Measuring the dynamics of gels

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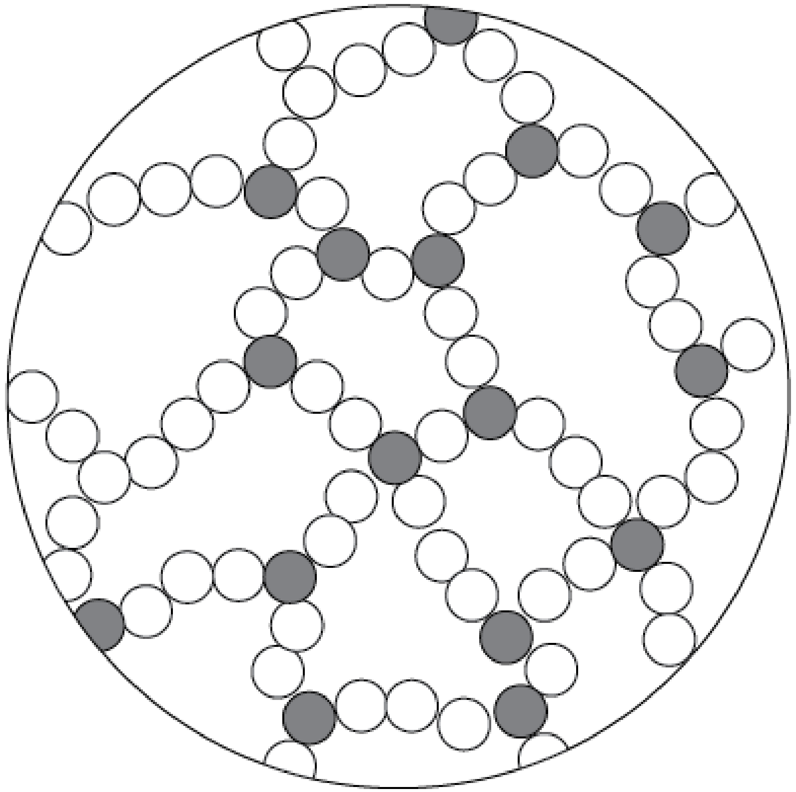
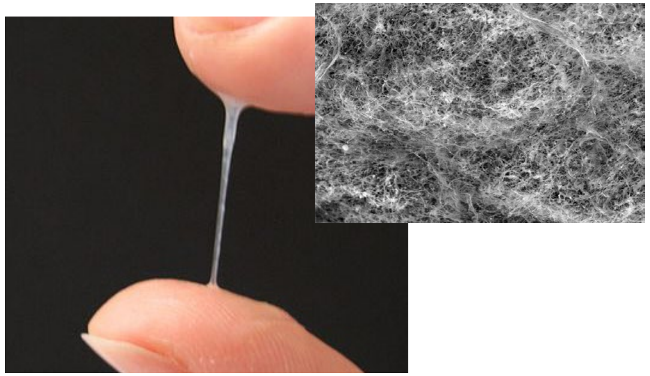
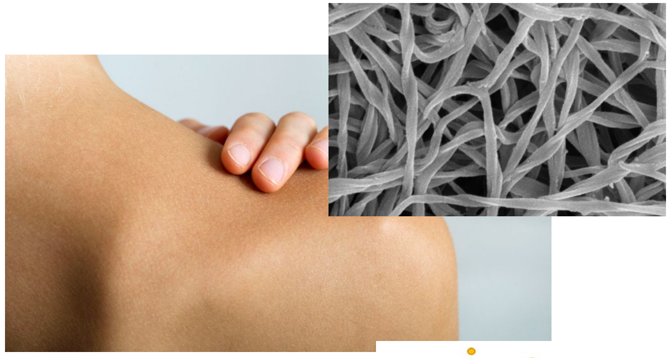
Gels



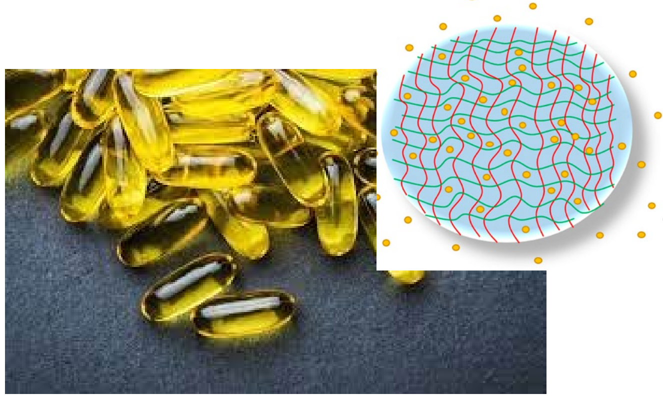
Constituent material (polymer)
Swollen in solvent (water -> “hydrogel”)
Bound and forms a network through cross-links



Biological fluids
e.g. mucus, biofilm

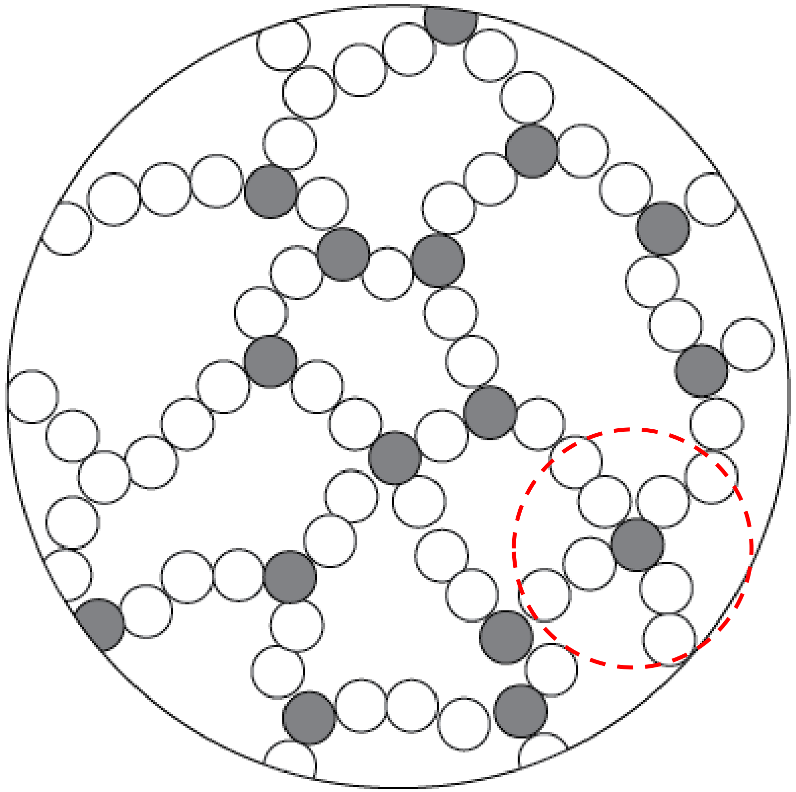


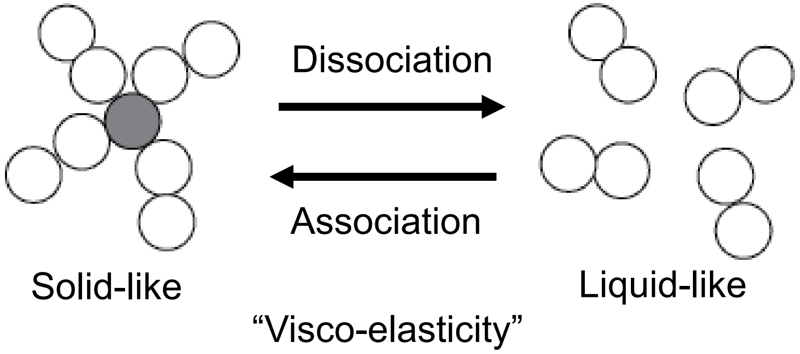
Biological materials
e.g. ECM, cells



Biomaterials
e.g. drug carriers, implants

Gels





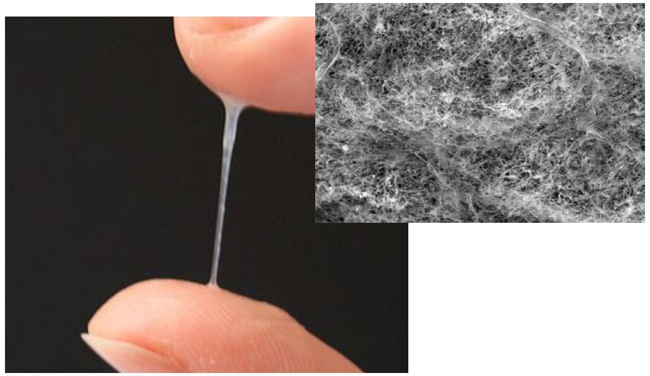
Dissociation

Association

Solid-like

Liquid-like

"Visco-elasticity"



Biological fluids
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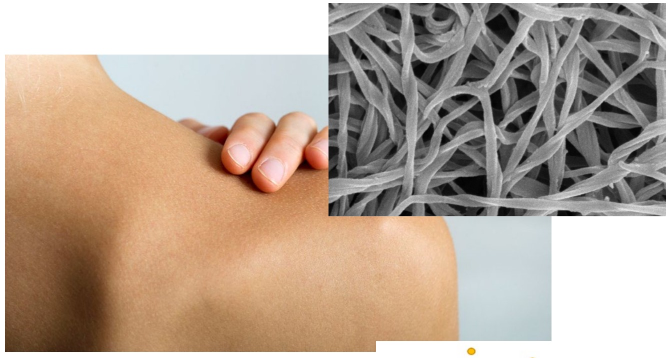
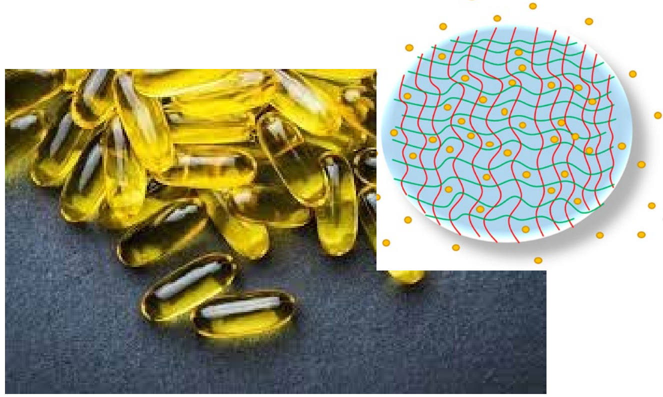


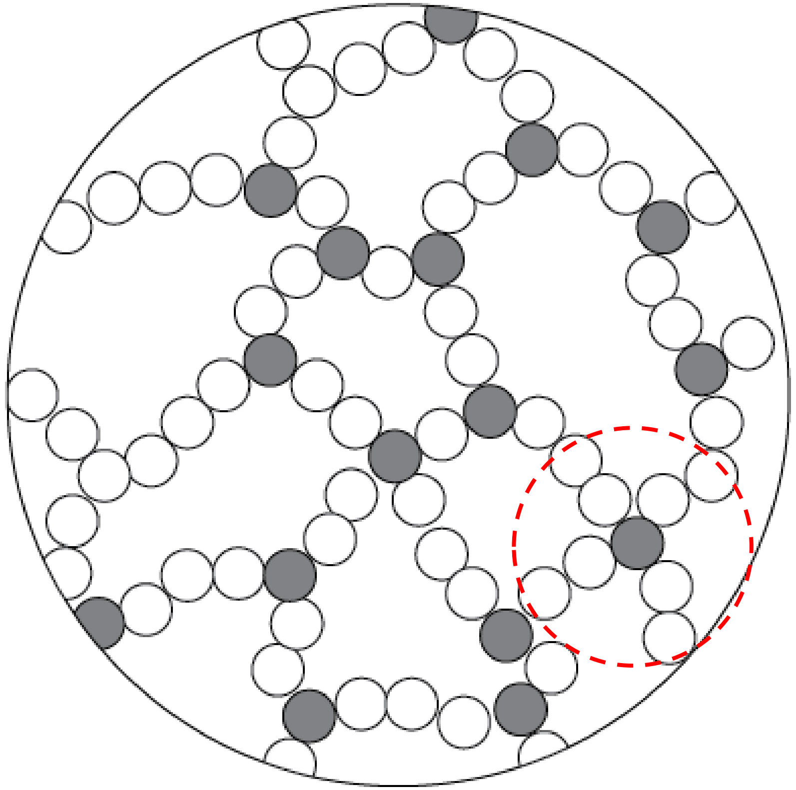
Biological materials
e.g. ECM, cells

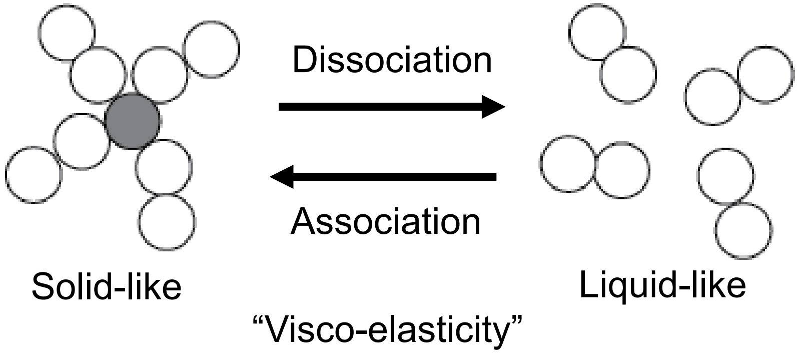


Biomaterials
e.g. drug carriers, implants

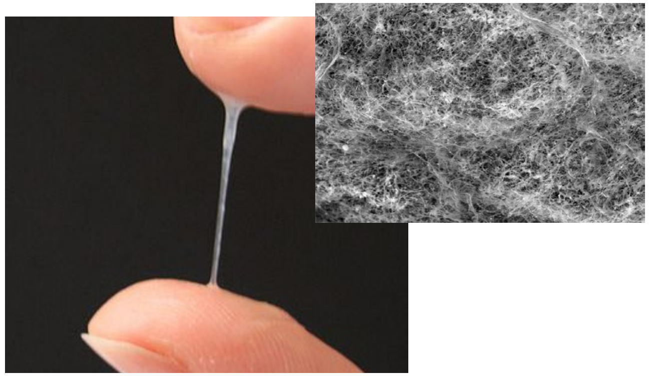
Gels

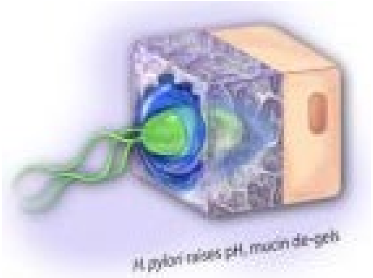
Dynamics and viscoelasticity dictate final properties of hydrogels



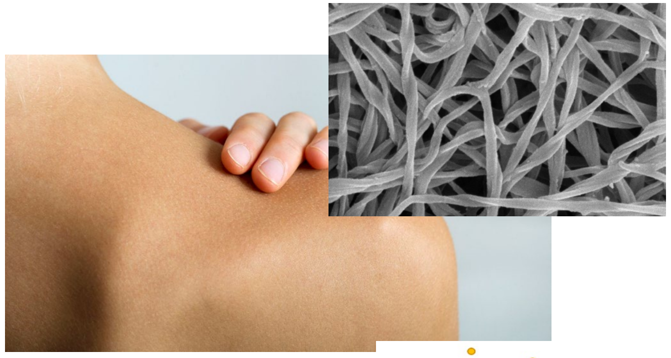


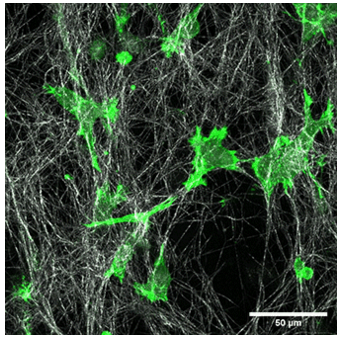
"Visco-elasticity"

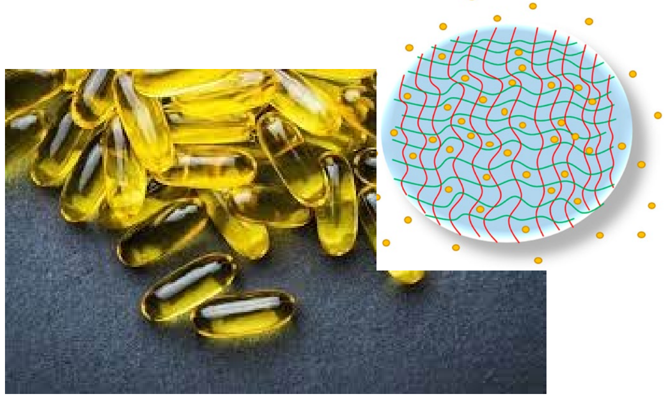


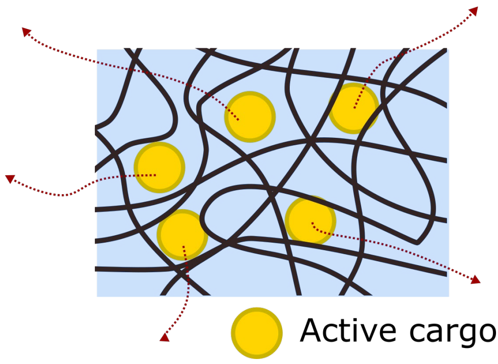


Bansil et al., PNAS 2009



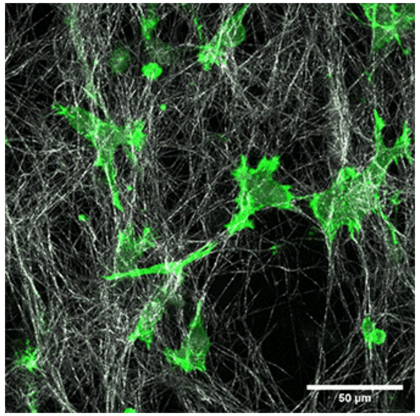




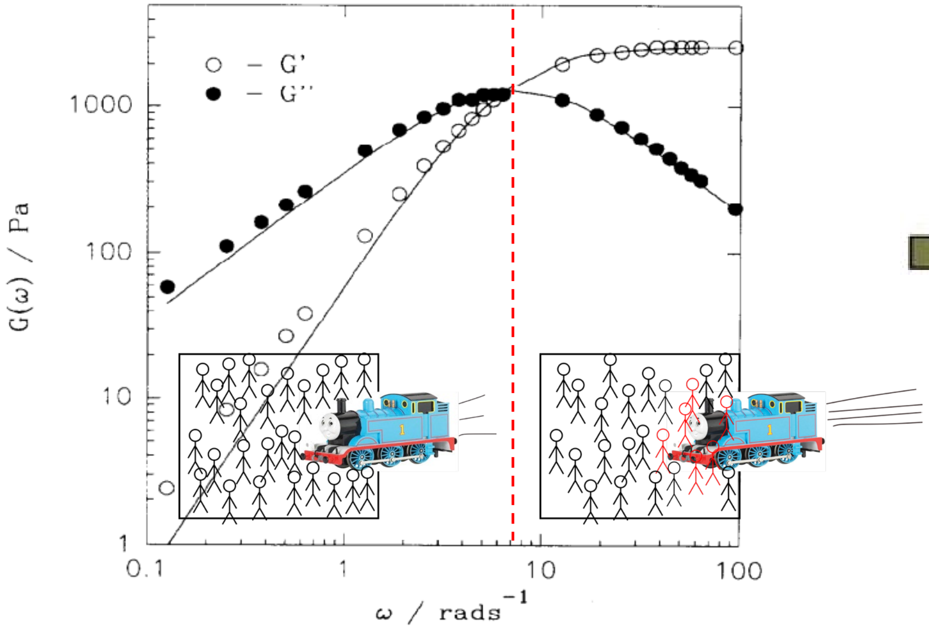


Viscoelasticity

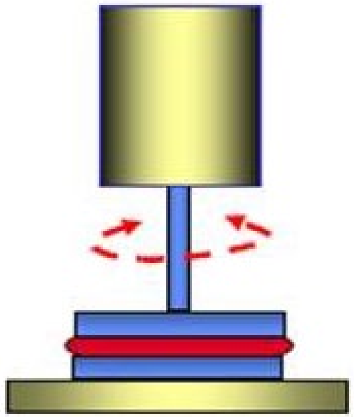
Why should we care?
Dynamics affects crucial biological function

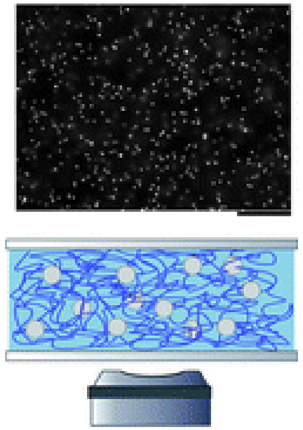


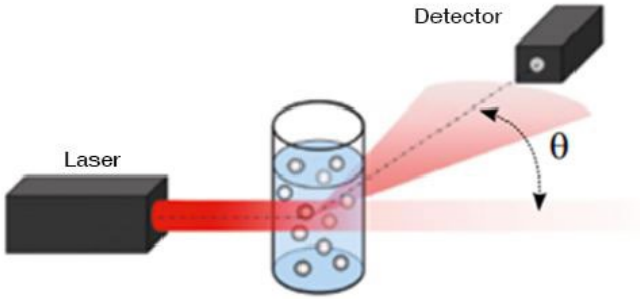
What is it?
Measure of elasticity, viscosity, and the dynamical timescale of the system



How do we measure it?
macrorheology, microrheology, …

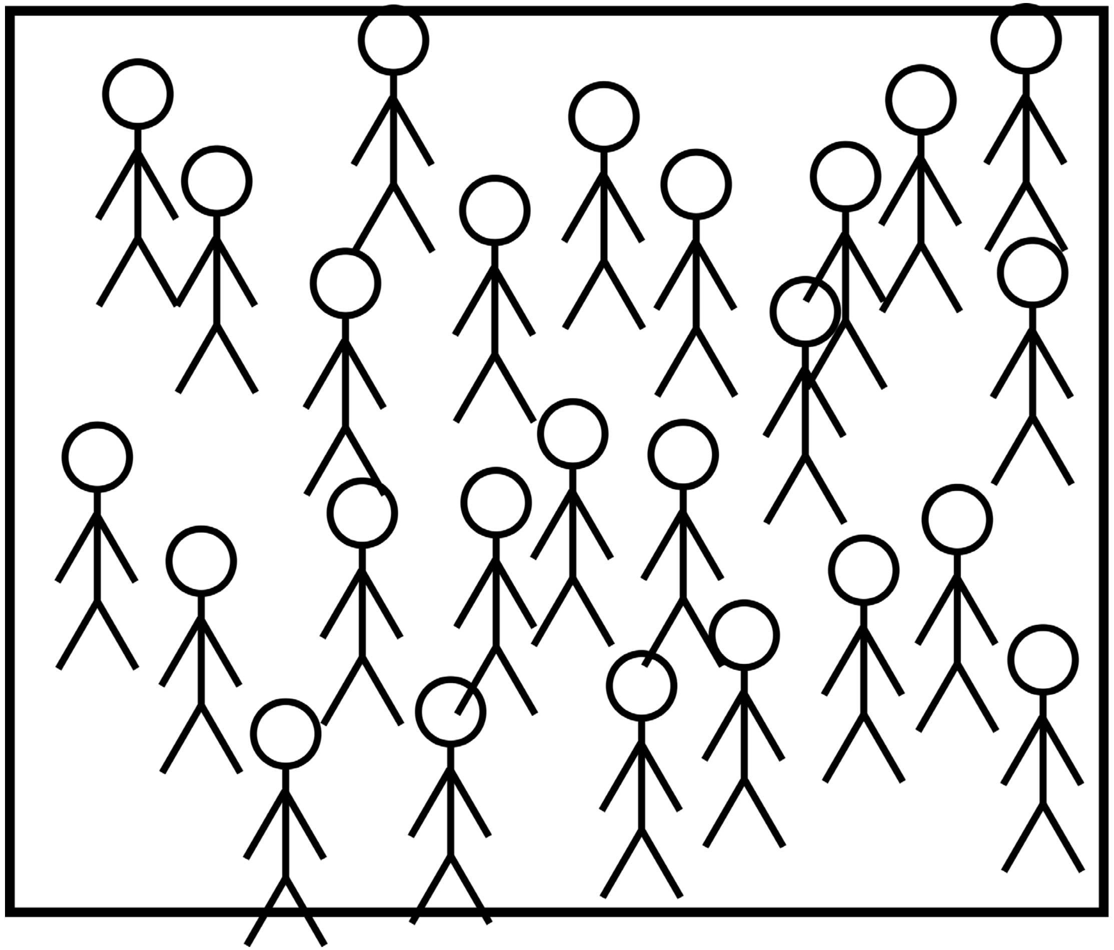




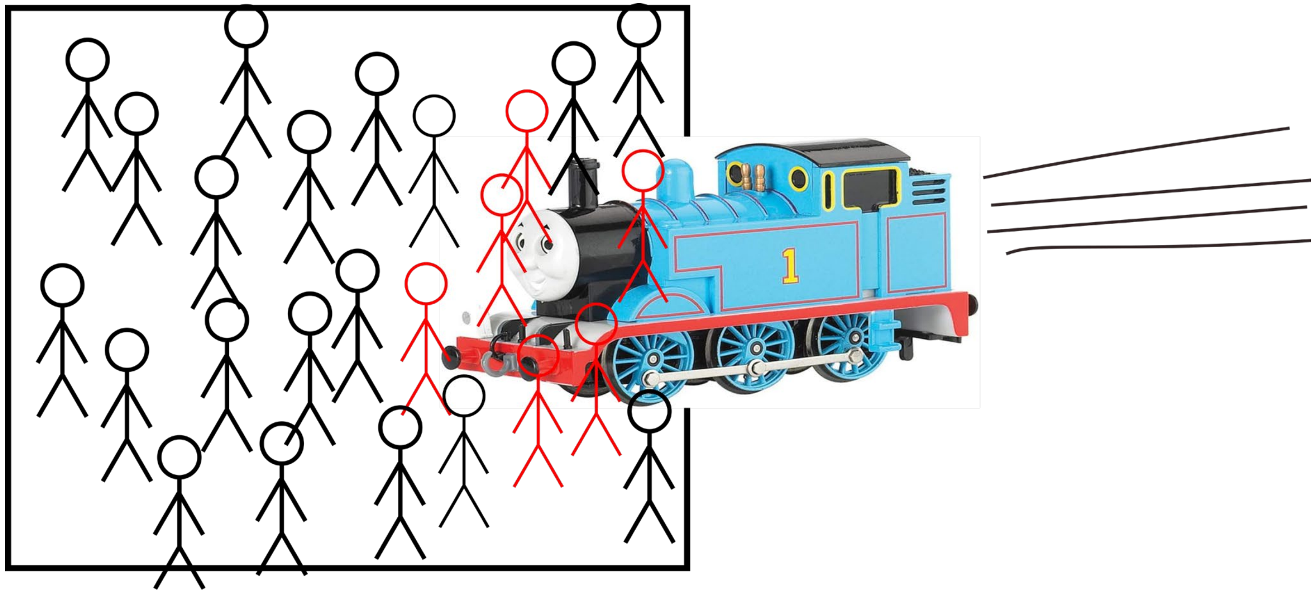


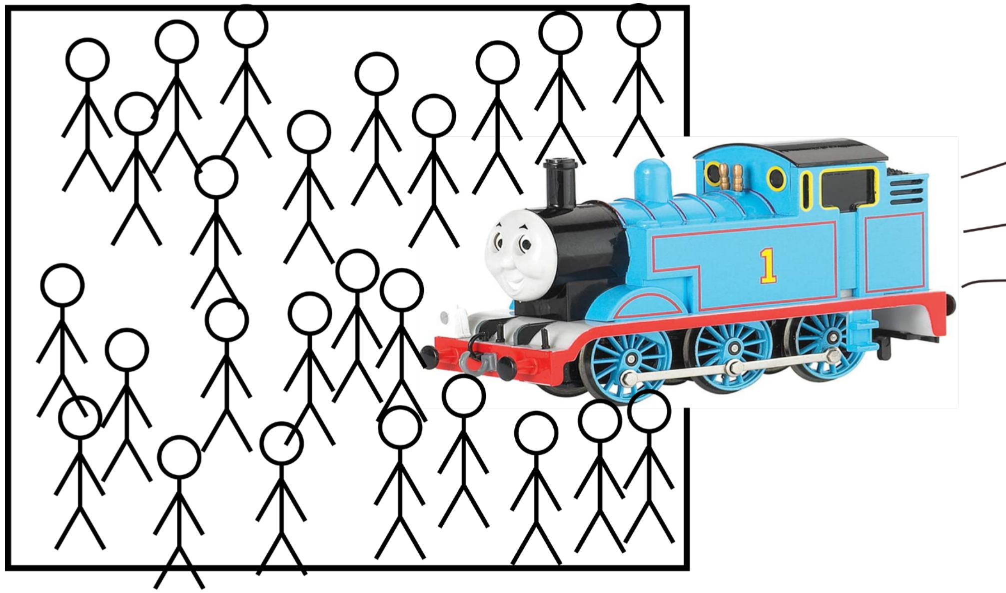
Viscoelasticity

The time/rate-dependent expression of viscosity and elasticity; ubiquitous in soft materials



Viscoelasticity – rate dependence

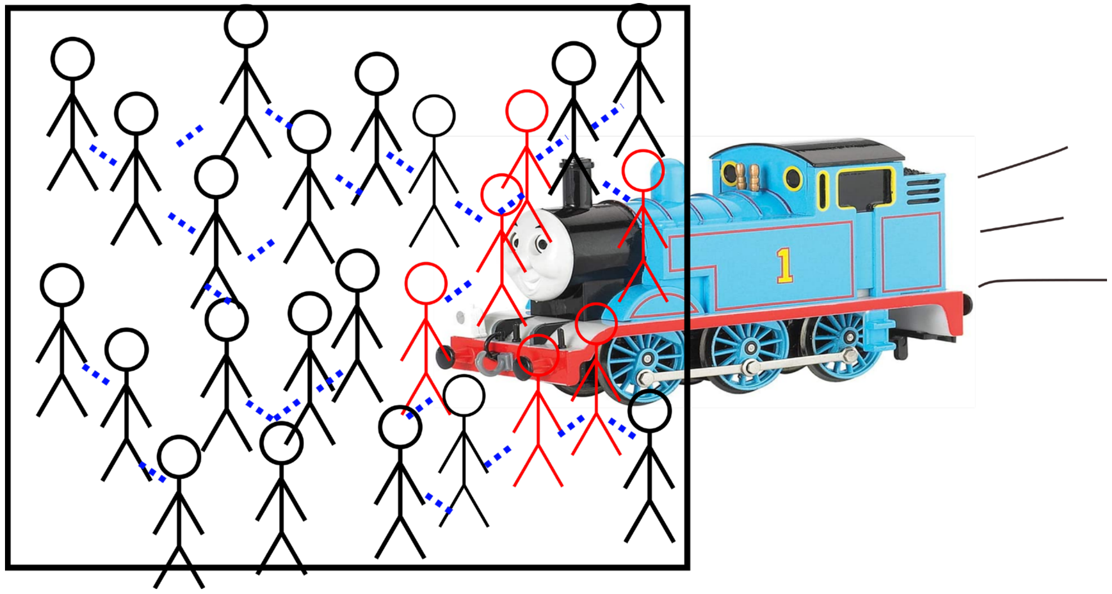




High strain rate
Insufficient time to rearrange
Solid-like “elastic” response

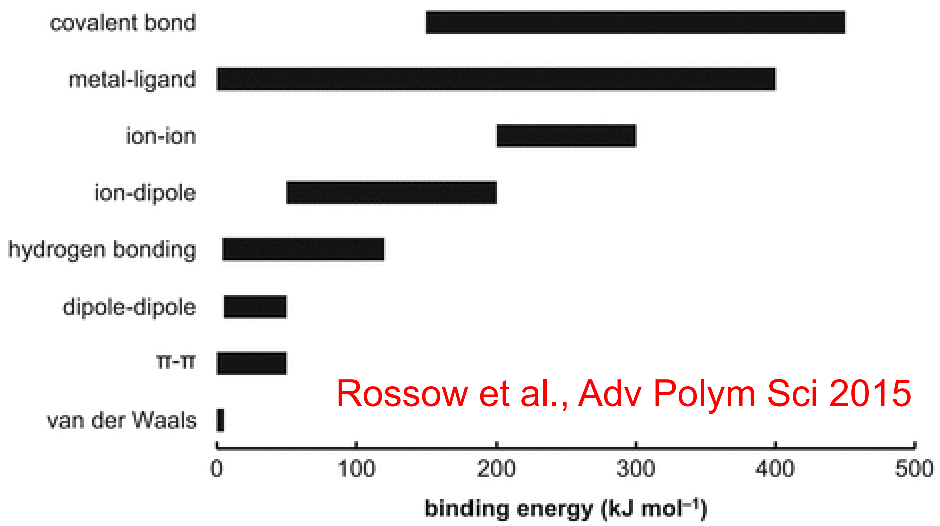
Low strain rate
Sufficient time to rearrange
No solid-like “elastic” response
Liquid-like “viscous” response

Viscoelasticity – rate dependence



Low strain rate, but with tethers
Rearrangement prevented by temporary bonds, such that slow strain rate can still elicit “elastic” response

Arrhenius equation: $\tau = \tau_0 \exp\left(-\frac{E}{kT}\right)$

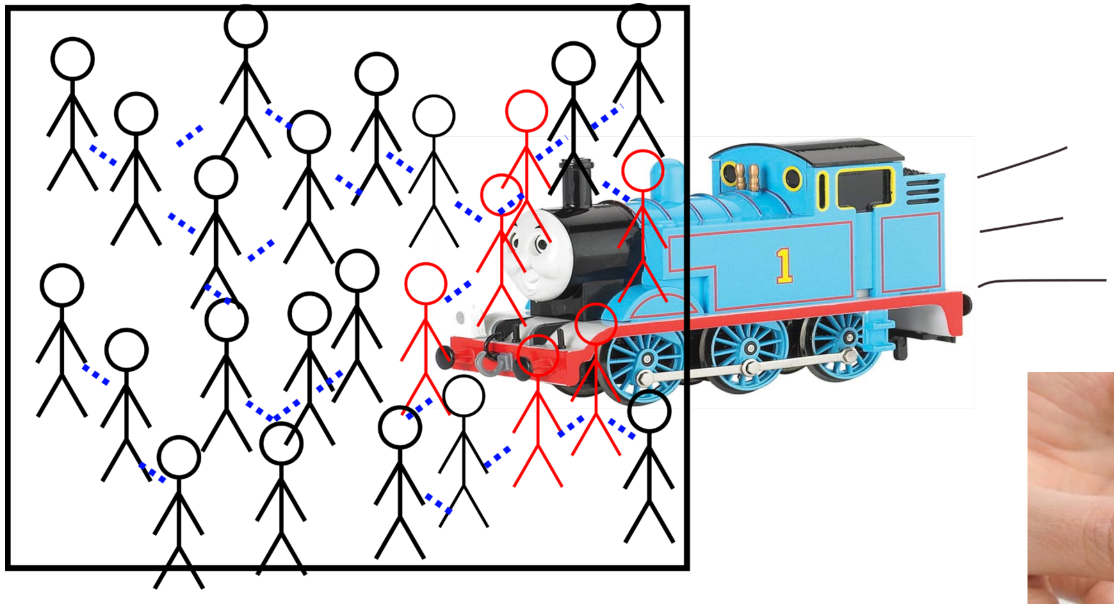


Rossow et al., Adv Polym Sci 2015

Bond lifetime controllable with different “physical bonds”, temperature, pH, …

Interplay of strain rate and bond lifetime dictates solid-like response

Viscoelasticity – rate dependence



Low strain rate, but with tethers
Rearrangement prevented by temporary bonds, such that slow strain rate can still elicit “elastic” response

Weissenberg number

$$\mathrm{Wi} = \text{strain rate} \times \text{relaxation time}$$
$$\mathrm{Wi} = \dot{\gamma}\tau$$

$\mathrm{Wi} > 1$

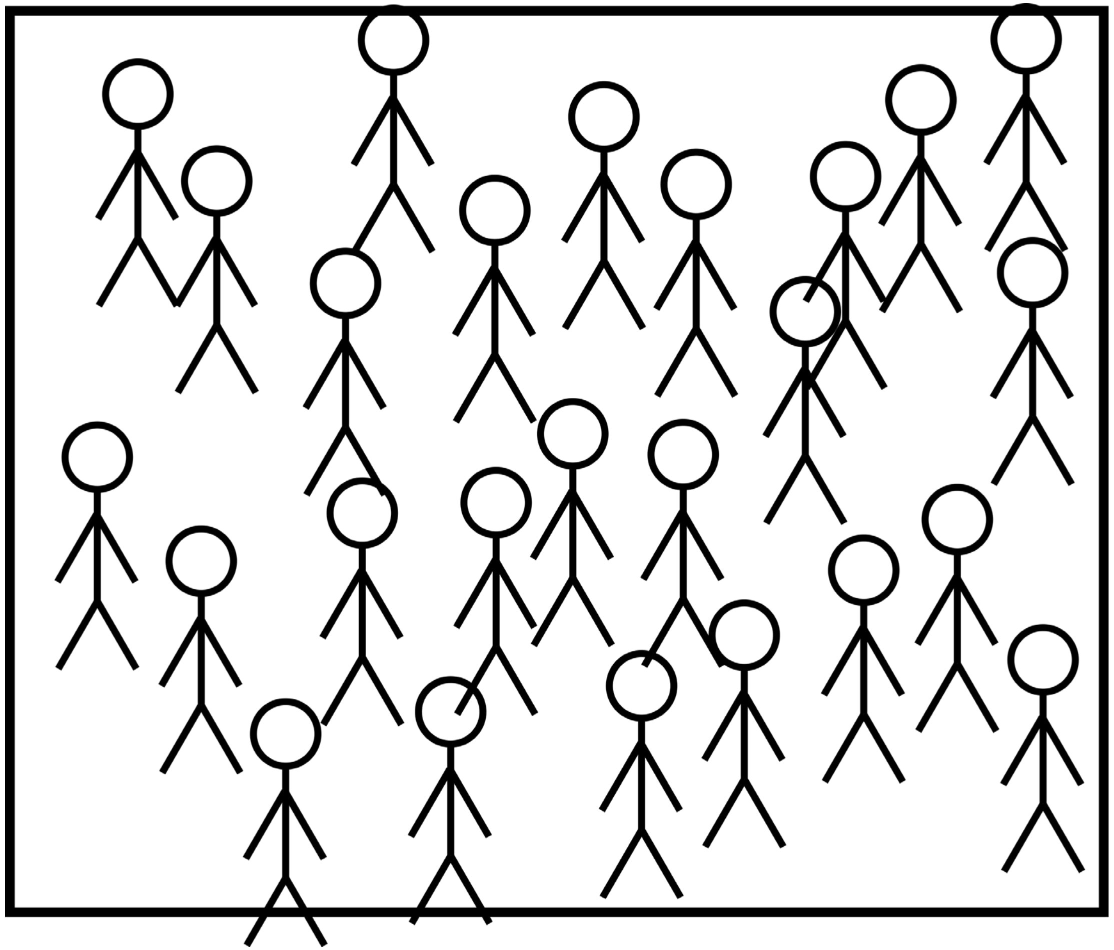
Solid-like behavior

$\mathrm{Wi} < 1$

Liquid-like behavior

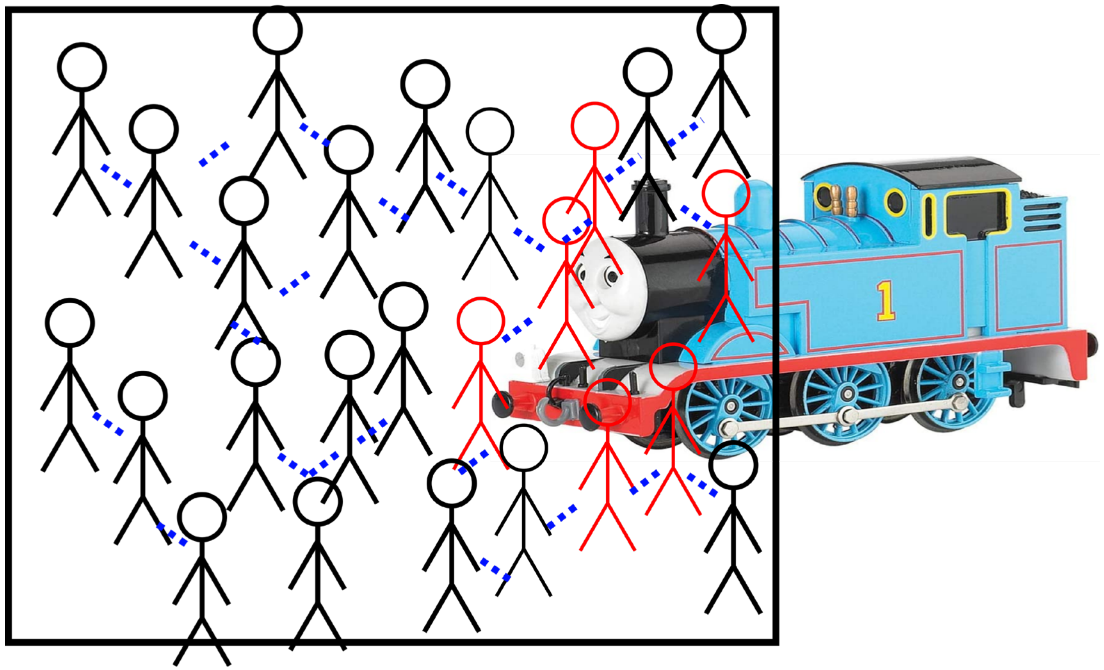
Probed by oscillatory shear measurements

Viscoelasticity

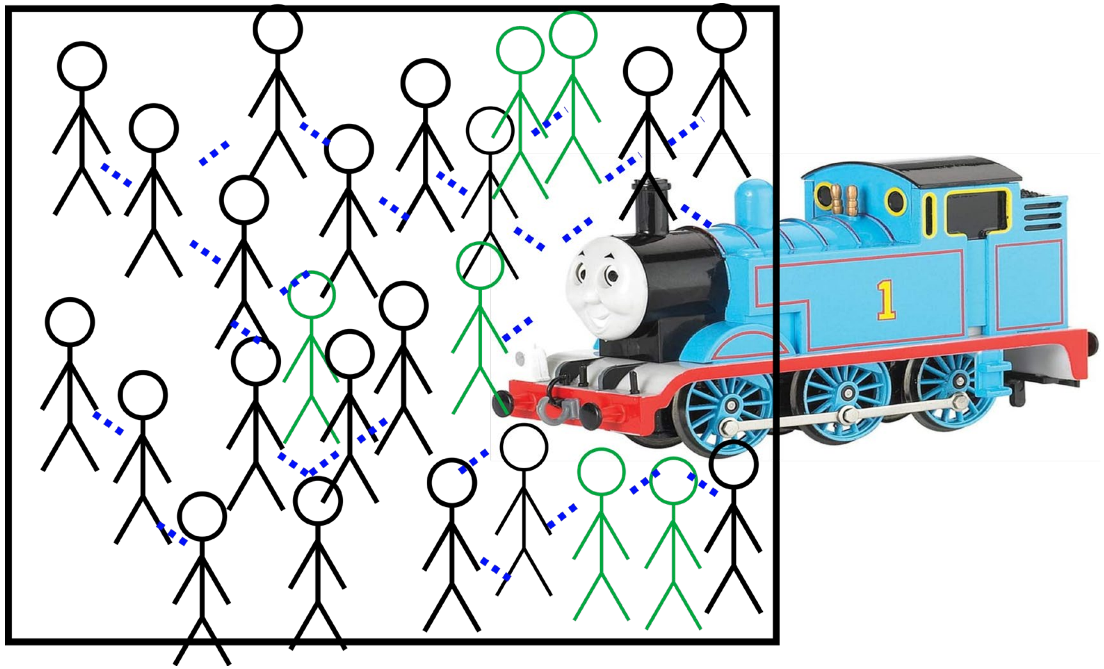


Crash and then park the train in the room…

Viscoelasticity – time dependence

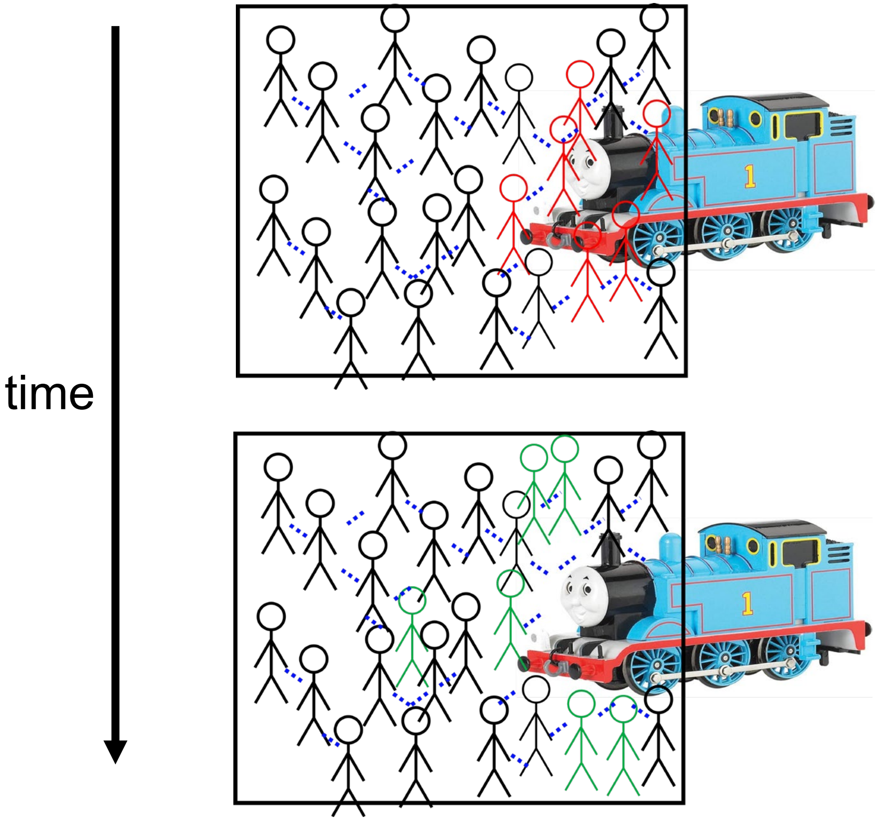


Short times after step deformation
“Elastic” response



Long times after step deformation
Enough time for rearrangements (at times longer than the “linker” lifetime)
Dissipation of the elastic response

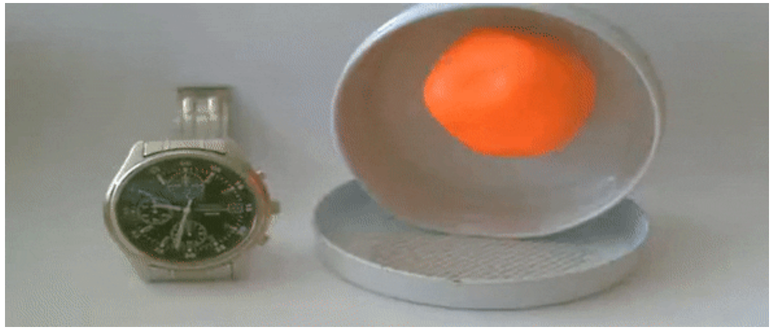
Viscoelasticity – time dependence



Probed by step strain measurements
“Stress relaxation”

Deborah number

$$\mathrm{De} = \frac{\textit{relaxation time}}{\textit{observation time}}$$



$\mathbf{De > 1}$

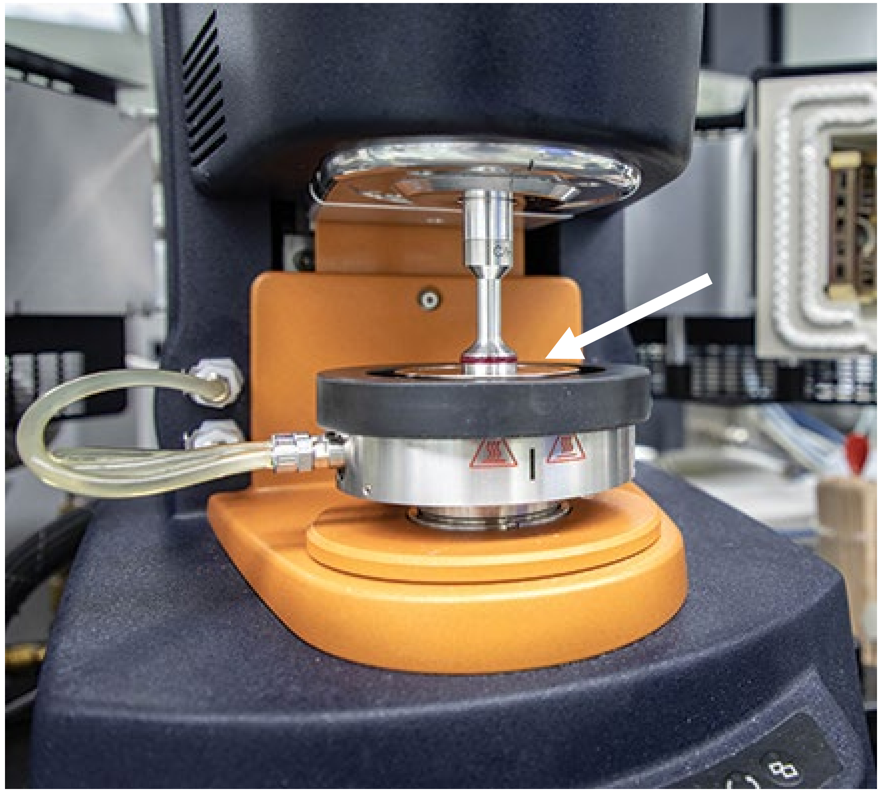
Solid-like behavior

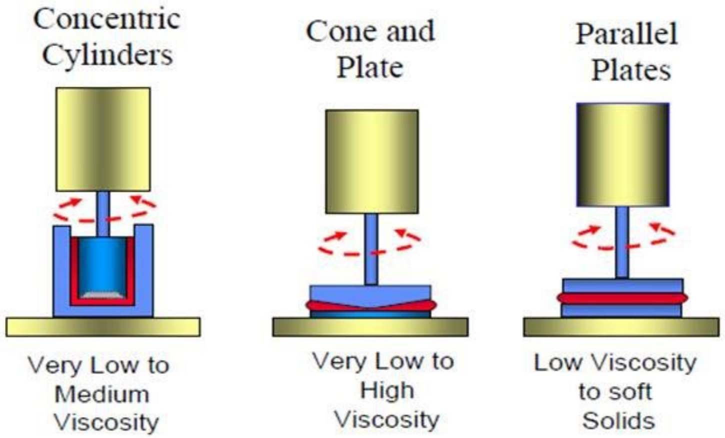
$\mathbf{De < 1}$

Liquid-like behavior

Rheology to study viscoelasticity

The most conventional approach: Use a rheometer





Concentric Cylinders

Very Low to Medium Viscosity

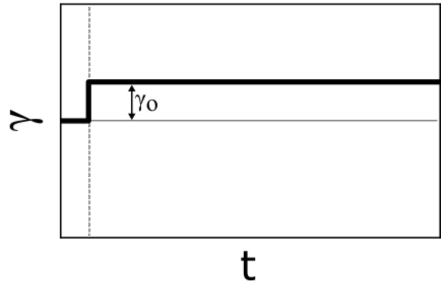
Cone and Plate

Very Low to High Viscosity

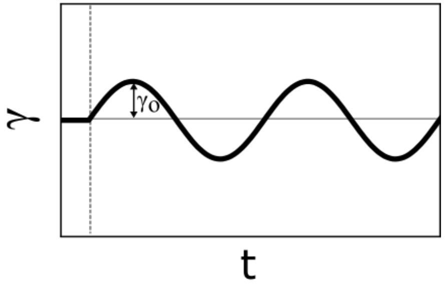
Parallel Plates

Low Viscosity to soft Solids

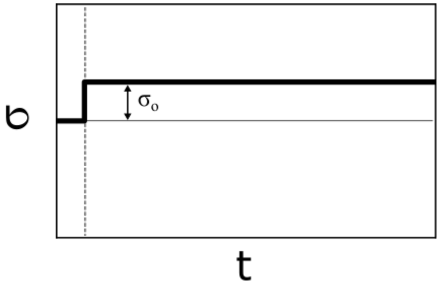
Typical sample volume:
60 μL ~ 1 mL



Step Strain



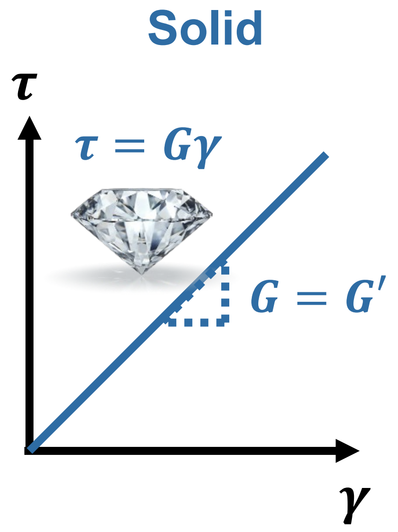
Oscillatory Strain

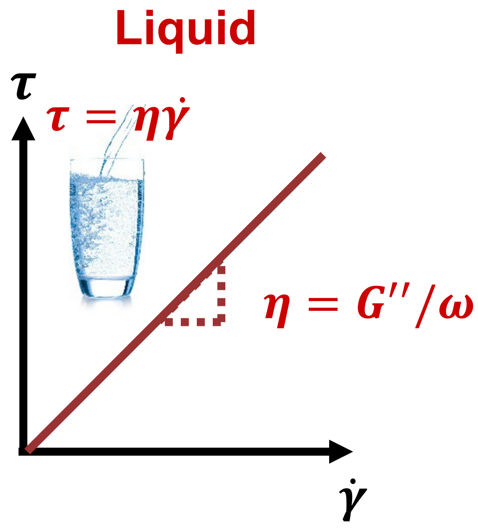


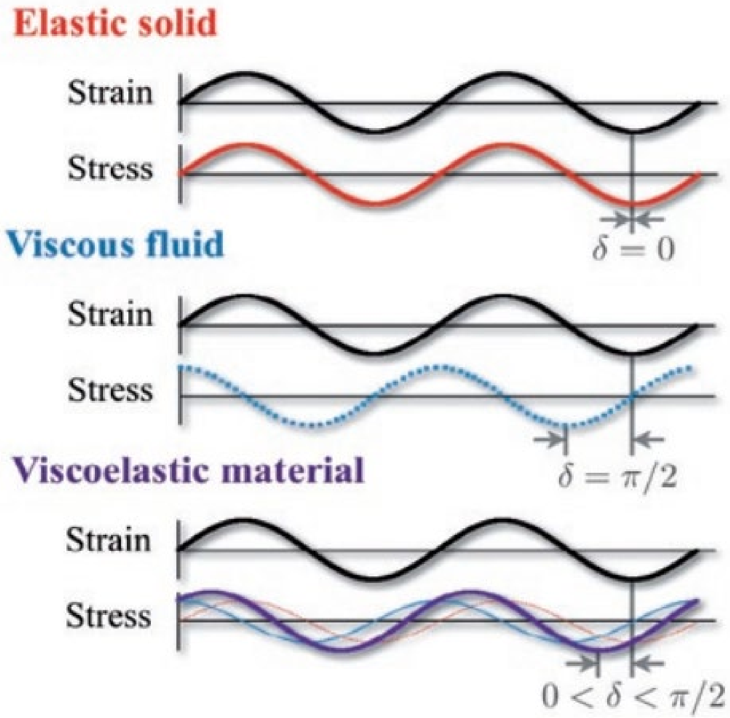
Creep

Useful textbook: “Rheology” by Chris Macosko

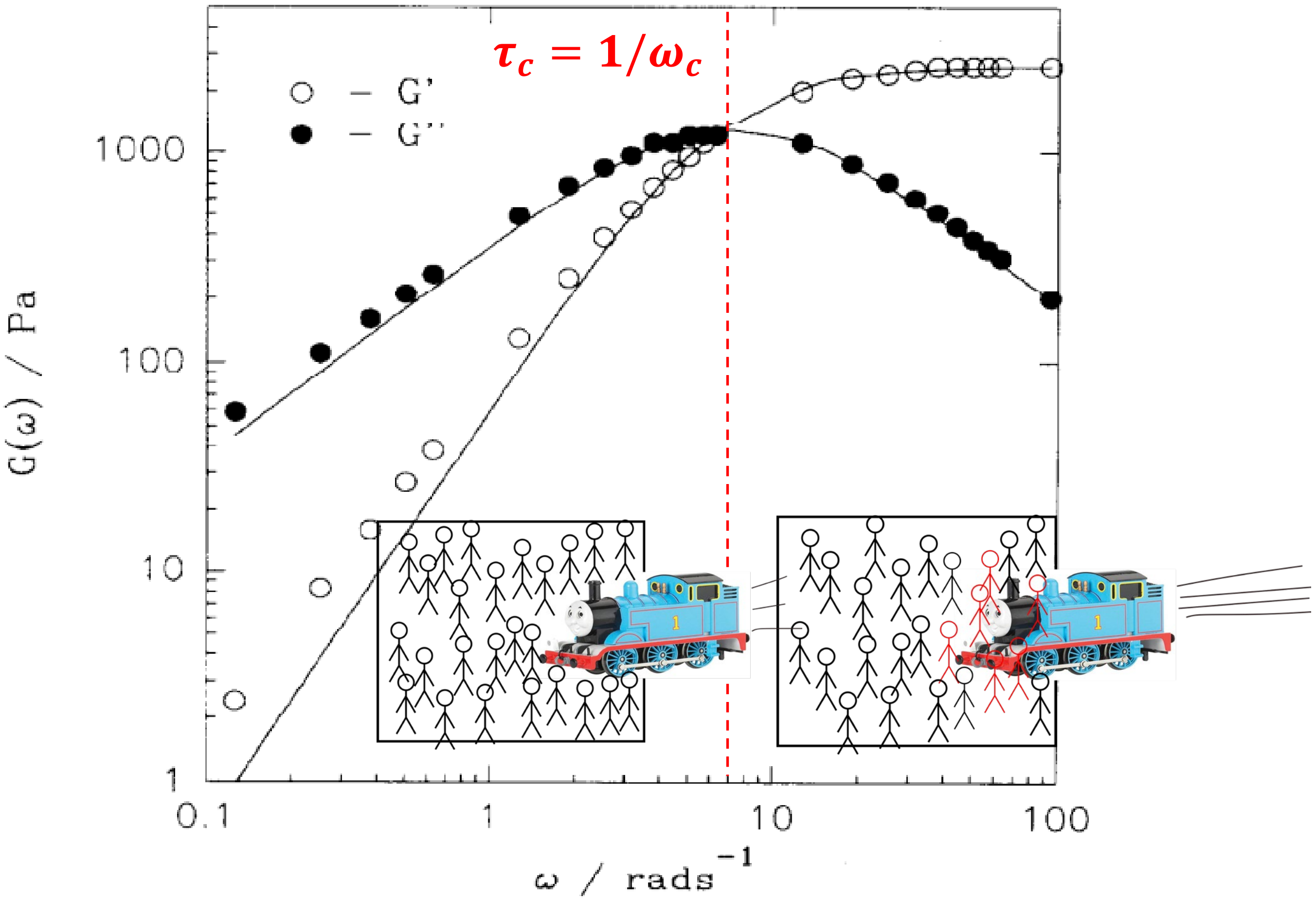
Rheology – Oscillatory Strain







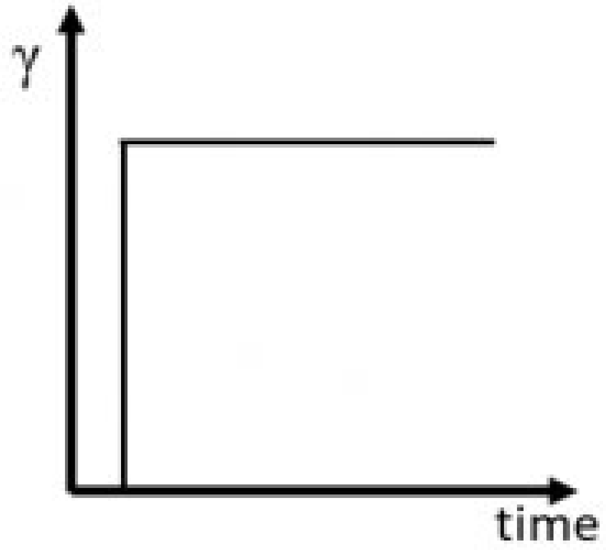
Example: Surfactant solution



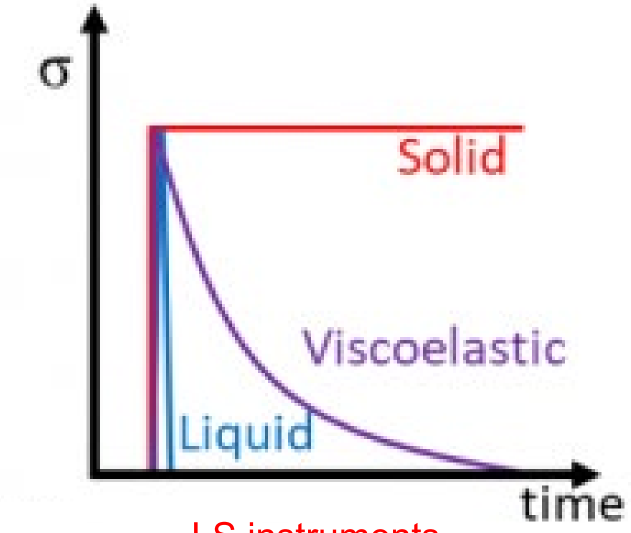
Annable, J Rheol 1993

Rheology – Step Strain

Impose step strain

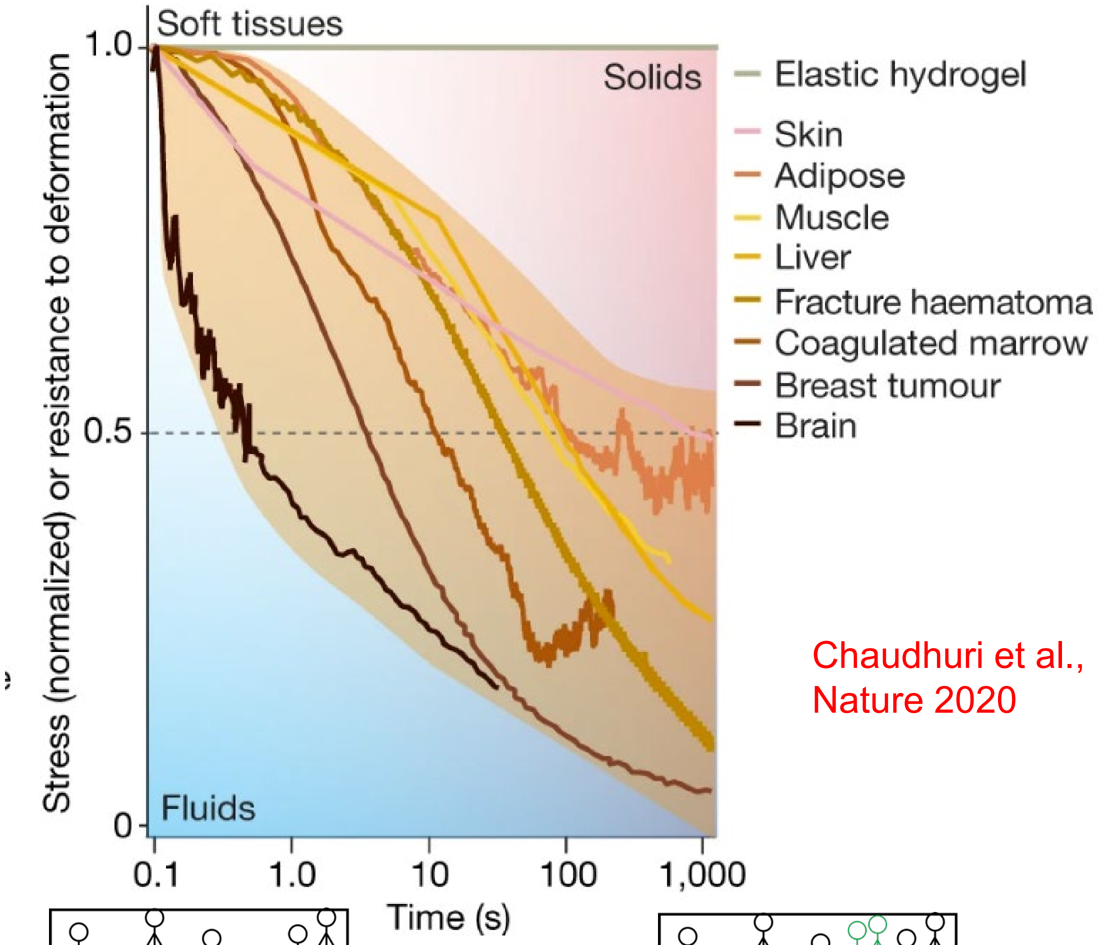


Observe time-dependent **stress relaxation**

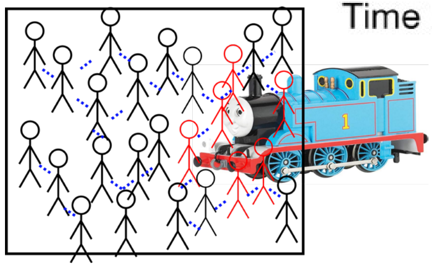


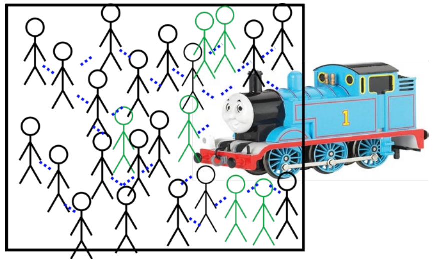
LS instruments

Example: Tissues

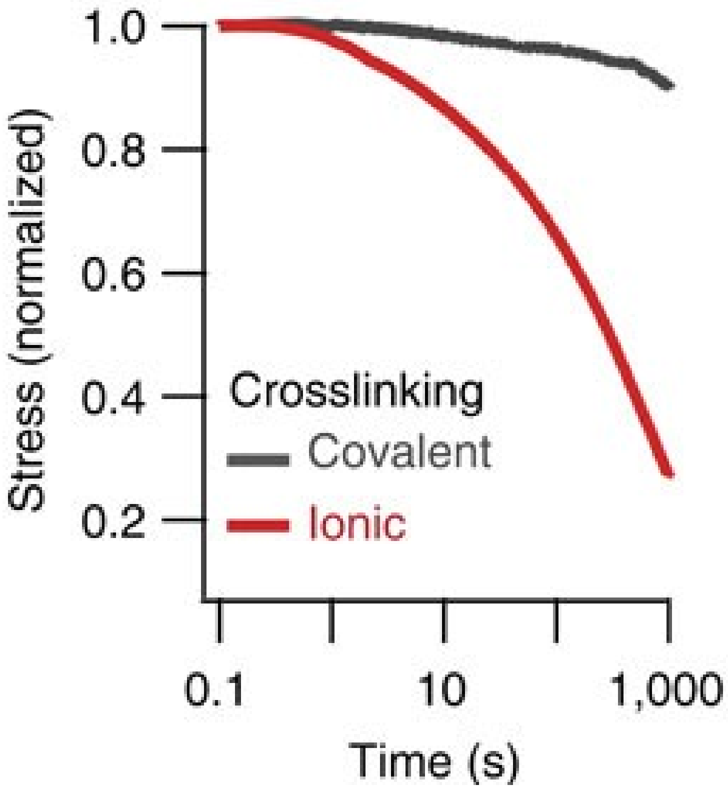


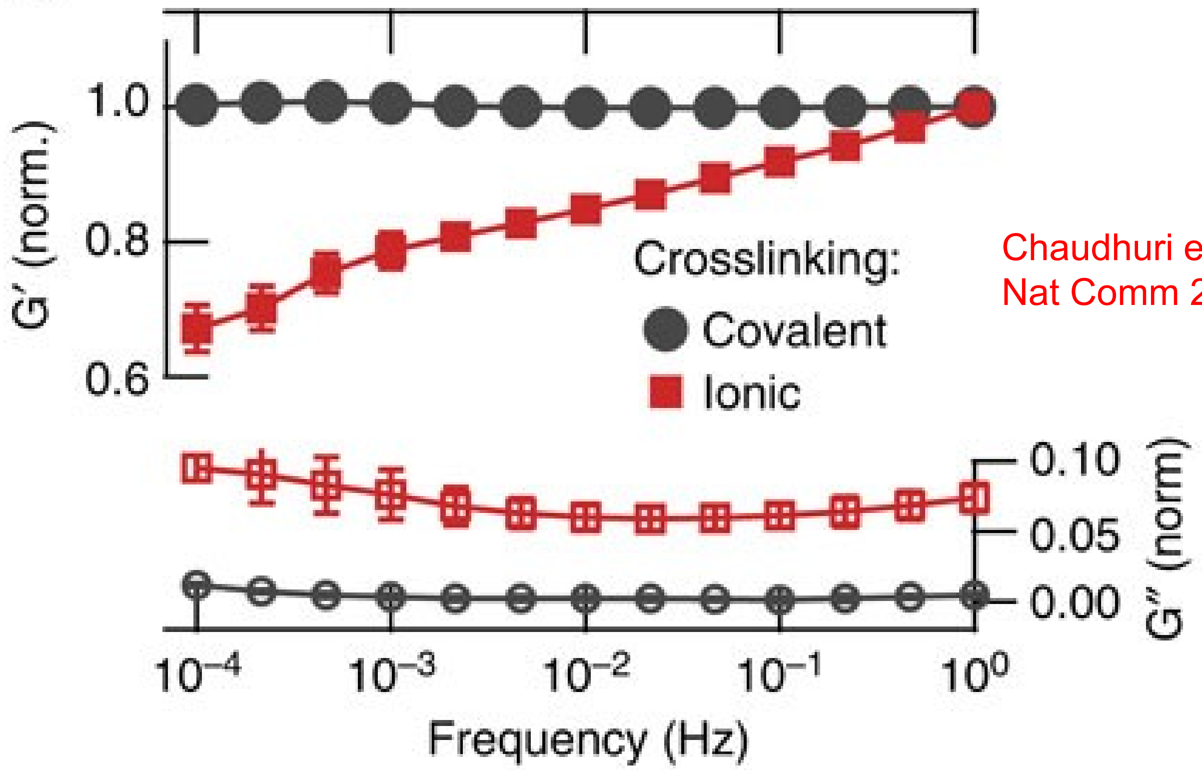
Chaudhuri et al., Nature 2020





Step Strain vs Oscillatory Shear



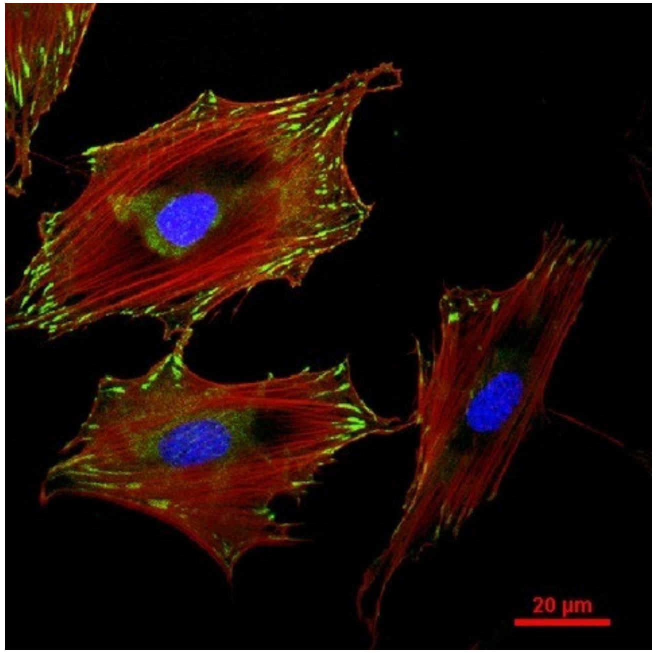


Chaudhuri et al.,
Nat Comm 2015

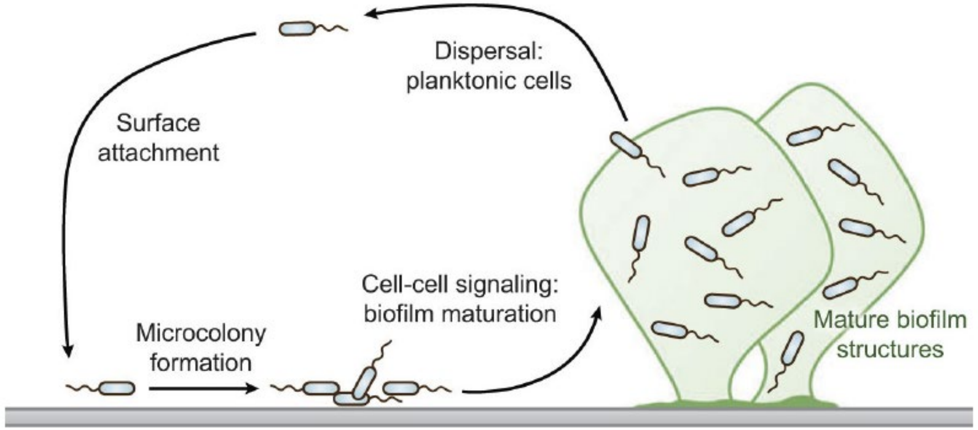
Step strain:
-> Less information, but larger window of time

Oscillatory shear
-> More information, but much slower / small window of time

Biological hydrogels are often small volume

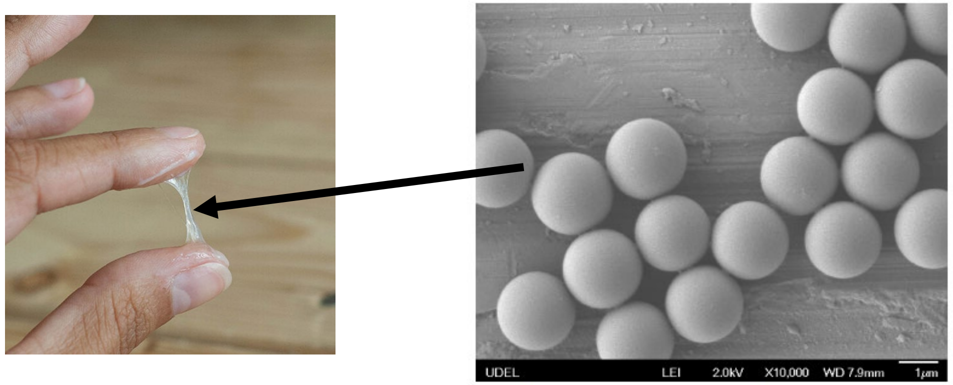


Conway institute

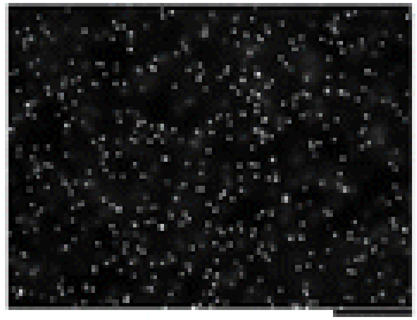


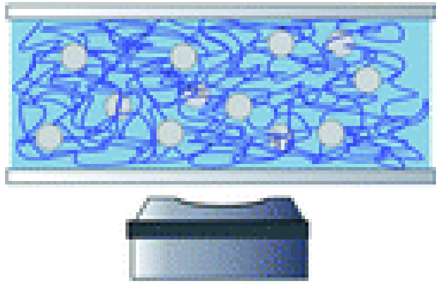
Small volumes in biological systems makes conventional rheology challenging

Solution: Microrheology



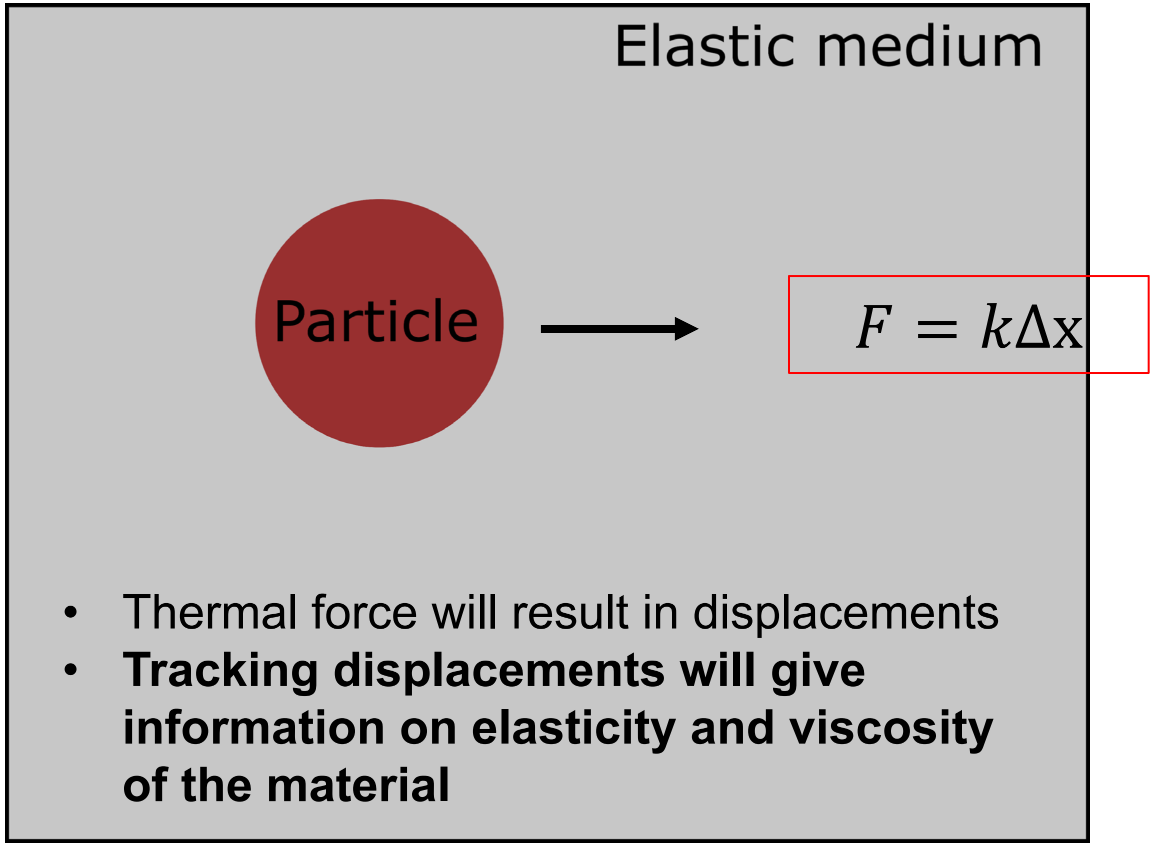
polystyrene microspheres





Observe fluctuation of (usually fluorescent) particles in hydrogel with a microscope

Eric Furst, SOR 2017 talk

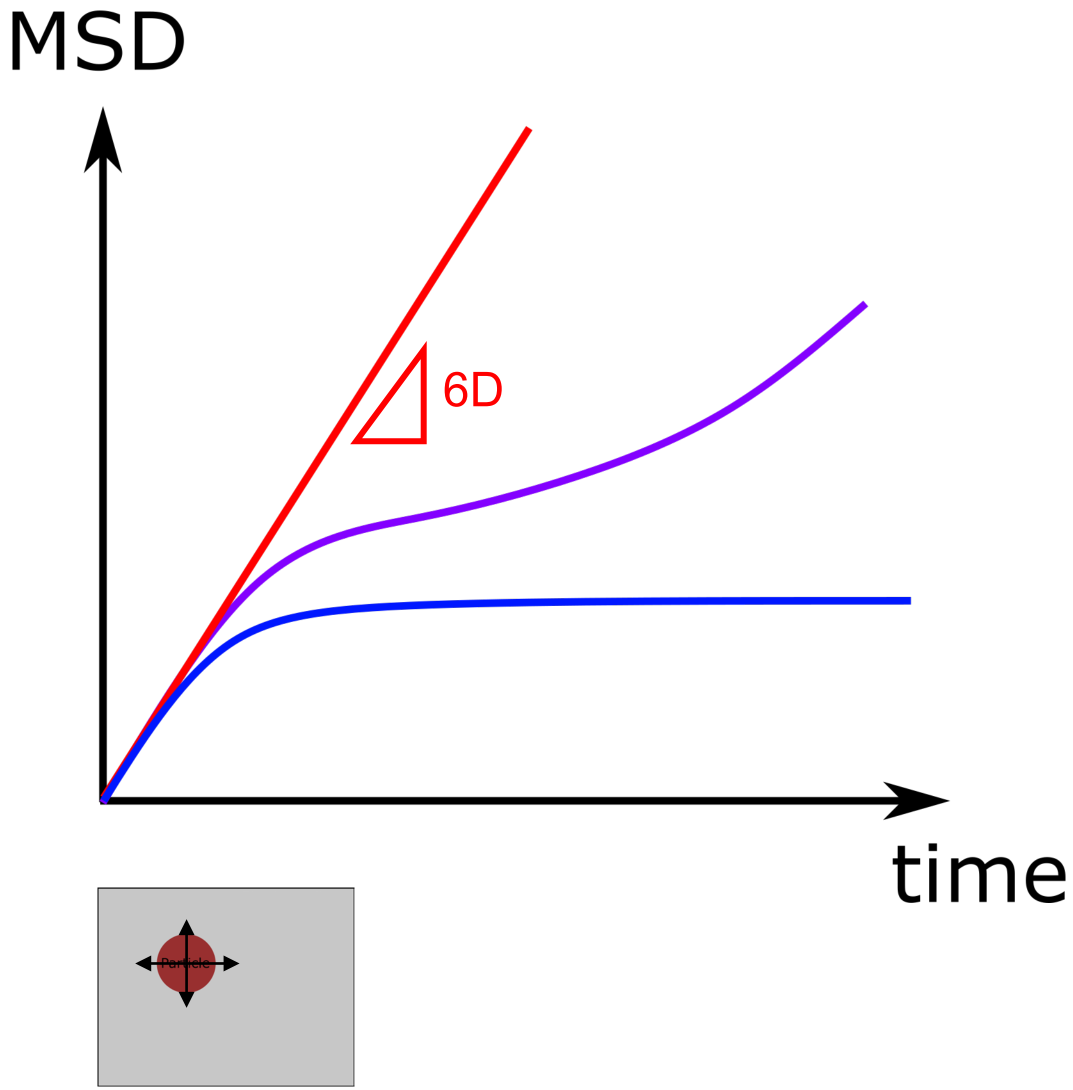


$$F = k\Delta x$$

- Thermal force will result in displacements
- **Tracking displacements will give information on elasticity and viscosity of the material**

Useful textbook: “Microrheology” by Eric Furst and Todd Squires

Microrheology



Liquid: MSD $\langle r^2(t)\rangle$ gives viscosity η

- ➢ Diffusivity D scales with η via the Stokes-Einstein theorem

$$D = \frac{k_B T}{6\pi\eta R}$$

Solid: MSD $\langle r^2(t)\rangle$ gives elasticity G

- ➢ Hooke's theorem

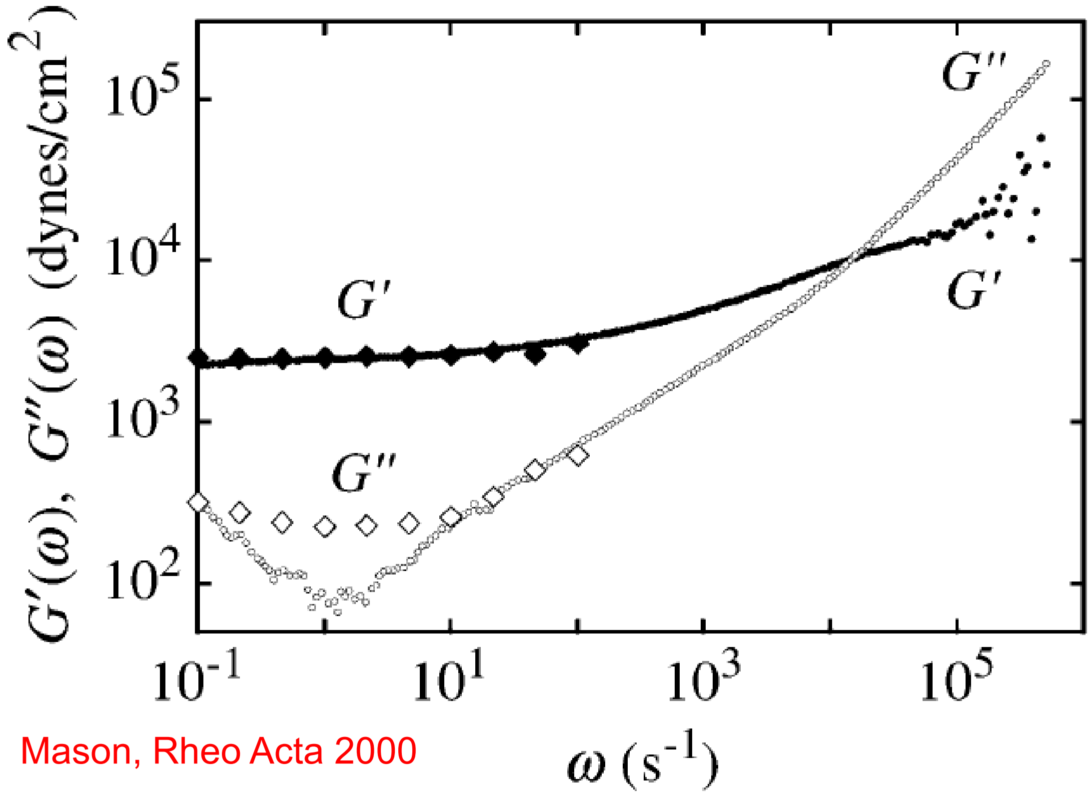
$$F = k\Delta x \Rightarrow kT = G\sqrt{\langle r^2(t)\rangle}$$

Viscoelastic: MSD interpolates between solid and liquid like response, gives $G^*(\omega)$

- ➢ Generalized Stokes-Einstein Theorem

$$\langle \Delta\tilde{r}^2(s)\rangle = \frac{k_B T}{\pi R s \tilde{G}^*(s)}$$

Microrheology – Example data



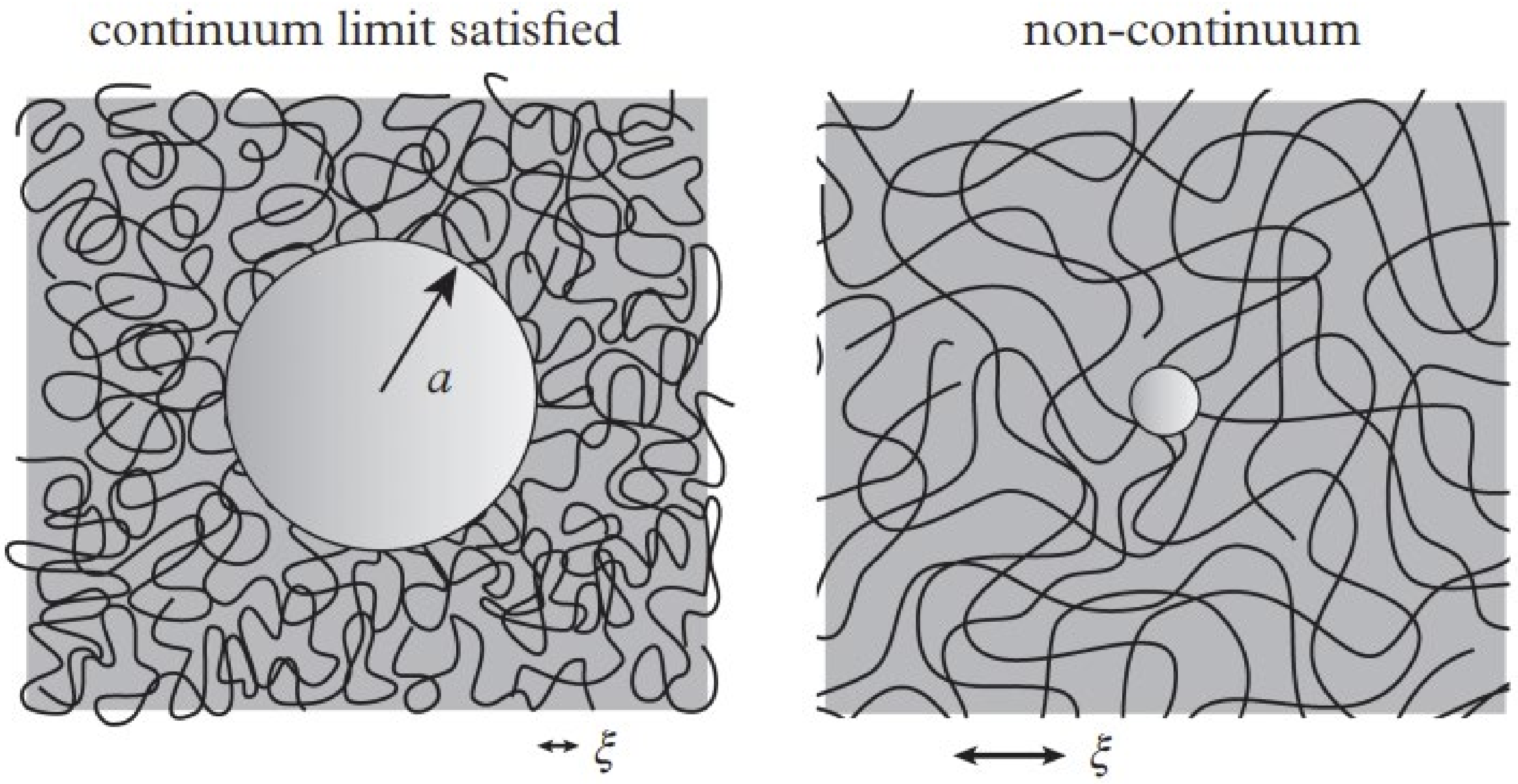
Mason, Rheo Acta 2000

Fig. 3 The frequency-dependent storage modulus, $G'(\omega)$, (solid symbols) and loss modulus, $G''(\omega)$, (open symbols) for the concentrated emulsion obtained from $\langle \Delta r^2(t) \rangle$ in Fig. 1 using the estimates for the generalized Stokes–Einstein equation, Eqs. (10) and (11) (small circles), and by mechanical measurements (large diamonds)

Pros:
Only need < 10 μL of sample
Can access higher frequencies
No sample contact / deformation

Cons:
Complicated analysis
Affected by structural heterogeneity
Can't examine stiff materials

Microrheology – Constraints and challenges



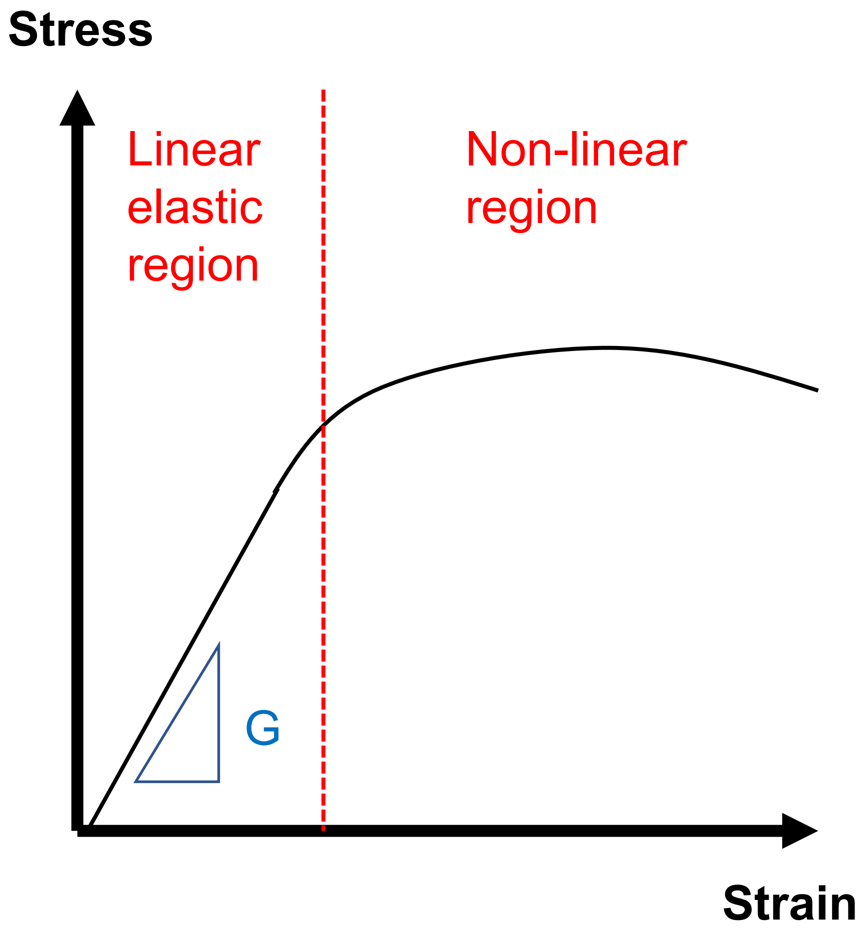
Requirement:

1) **Embedded particle must be bigger than the mesh size of the hydrogel***
2) **No particle-particle or particle-polymer interactions**

* Tracking dynamics of smaller particles can be a useful way to study mesh size

Additional readings: “Microrheology” by Eric Furst and Todd Squires
Mason and Weitz, PRL 1995
Gittes and Schnurr et al., Macromol 1997, PRL 1997
Squires and Mason, Ann Rev Fluid Mech 2010

Other forms of rheology to keep in mind

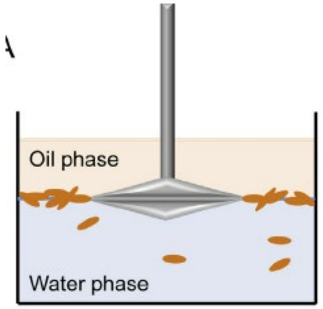


Scattering microrheology: Microrheology using light/x-ray scattering rather than microscopy – easier data analysis but requires more technical setup (see additional slides)

Dynamic mechanical analysis: Similar to rheometry but in normal direction rather than shear; more useful for stiff materials (MPa~GPa)

Interfacial rheology: Useful for measuring interface mechanics

Example: Measuring surfactant layer viscoelasticity in oil-water emulsions



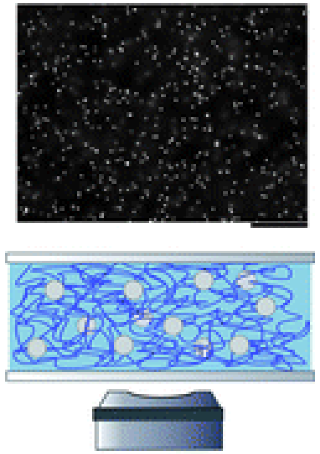
Ruhs et al., Colloids and surfaces B 2014

Non-linear rheology: Extensional rheology, large-amplitude oscillatory shear, steady shear, active microrheology, …

Summary of “current” techniques

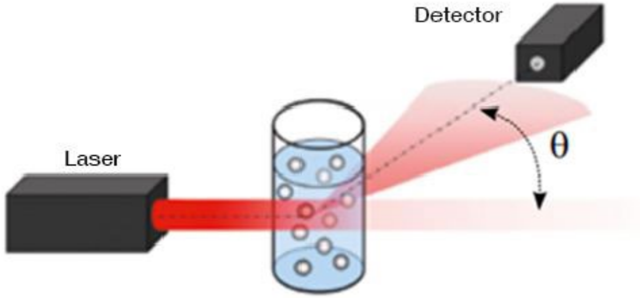
Particle tracking

- ~10^2 Pa
- ~ 1 uL vol
- Higher frequency
- Difficult analysis
- Sample heterogeneity



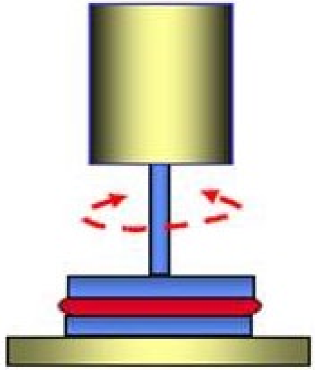
Scattering

- ~10^4 Pa
- 10~200 uL vol
- Higher frequency
- Specialized instrument
- Sample heterogeneity



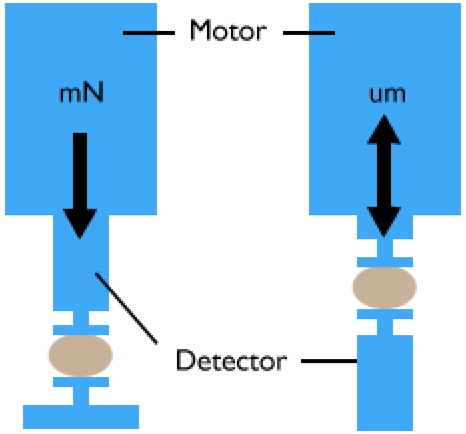
Rheometer

- ~10^6 Pa
- Established method
- Easy features like T control
- 50 ~ 500 uL vol
- Requires mechanical deformations



Dynamic mechanical analyzer

- 10^6~10^9 Pa
- 500 ~ 5000 uL vol
- Hard samples only
- Required mechanical deformations

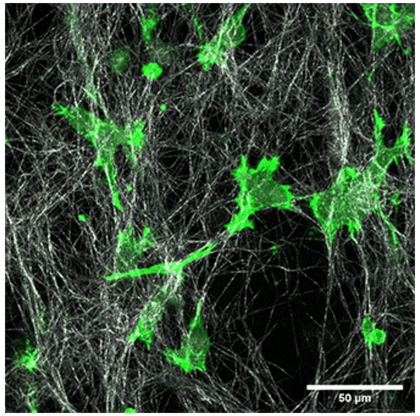


Sample volume, Modulus

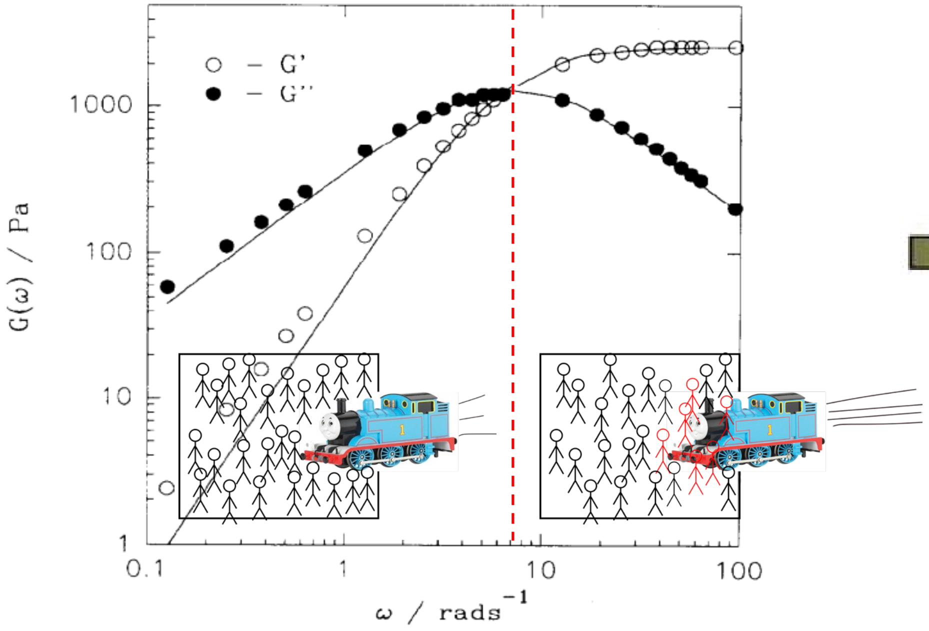
Schulz and Furst, Soft Matter 2012
Sepe, Dynamical Mechanical Analysis for Plastics Engineering 1998

Viscoelasticity

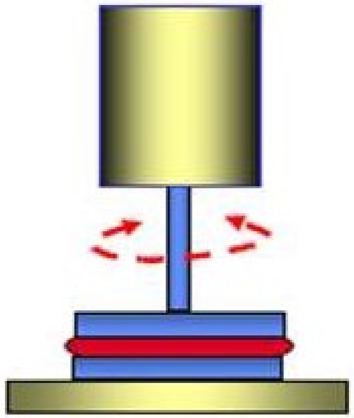
Why should we care?
Dynamics affects crucial biological function

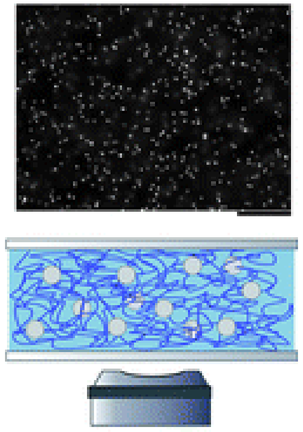


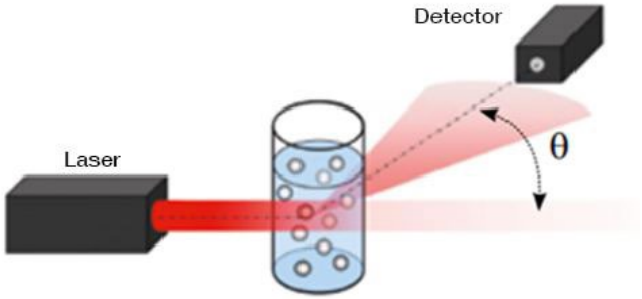
What is it?
Measure of elasticity, viscosity, and the dynamical timescale of the system



How do we measure it?
macrorheology, microrheology, …







Additional slides

Microrheology – theoretical background

$$\eta^*(\omega) = i\omega G^*(\omega)$$

$$\mathcal{L}(V(t)) = \tilde{V}(s)$$

The Stokes-Einstein theorem:

$$D = \frac{k_B T}{\xi} = \frac{k_b T}{6\pi\eta a}$$

Einstein: Diffusivity related to hydrodynamic resistance

Stokes: Diffusivity related to viscosity

Langevin equation (variation of Newton's law of motion):

$$m\frac{dV(t)}{dt} = f_R(t) - \int_0^t \xi(t-t')V(t')dt'$$

Laplace transform -> Multiply by $V(0)$ to obtain velocity autocorrelation function (to obtain the MSD in complex space) which can be related to ξ

$$\langle V(0)\tilde{V}(s)\rangle = \frac{s^2}{2}\langle\Delta\tilde{r}^2(s)\rangle = \frac{nk_BT}{\tilde{\xi}(s)}$$

The Generalized Stokes-Einstein Relation (GSER)

$$\boxed{\langle\Delta\tilde{r}^2(s)\rangle = \frac{nk_BT}{3\pi a s\tilde{G}^*(s)}}$$

User-friendly adaptation

$$\tilde{G}^*(s) = \frac{nk_BT}{3\pi a\langle r^2(t)\rangle\Gamma\left(1 + \frac{dln\langle r^2(t)\rangle}{dlnt}\right)}$$

$$\boldsymbol{G^*(\omega) = \mathcal{L}^{-1}(\tilde{G}^*(s))}$$

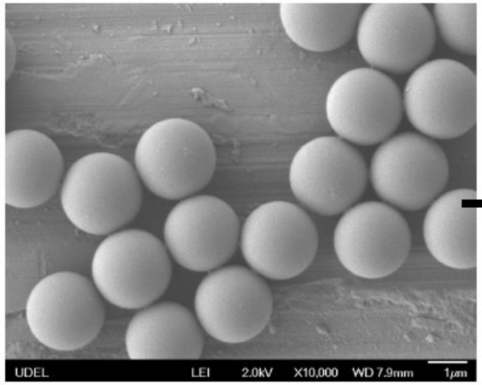
Mason and Weitz, PRL 1995
Squires and Mason, Ann Rev Fluid Mech 2010
Mason, Rheo Acta 2000

Microrheology via scattering

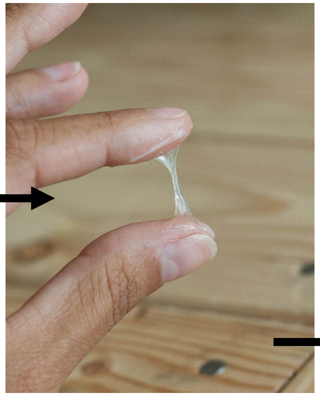
Idea: Instead of tracking individual particles, track the motion of ensemble of particles in reciprocal space
Method: Diffusing wave spectroscopy (DWS), X-ray photon correlation spectroscopy (XPCS)

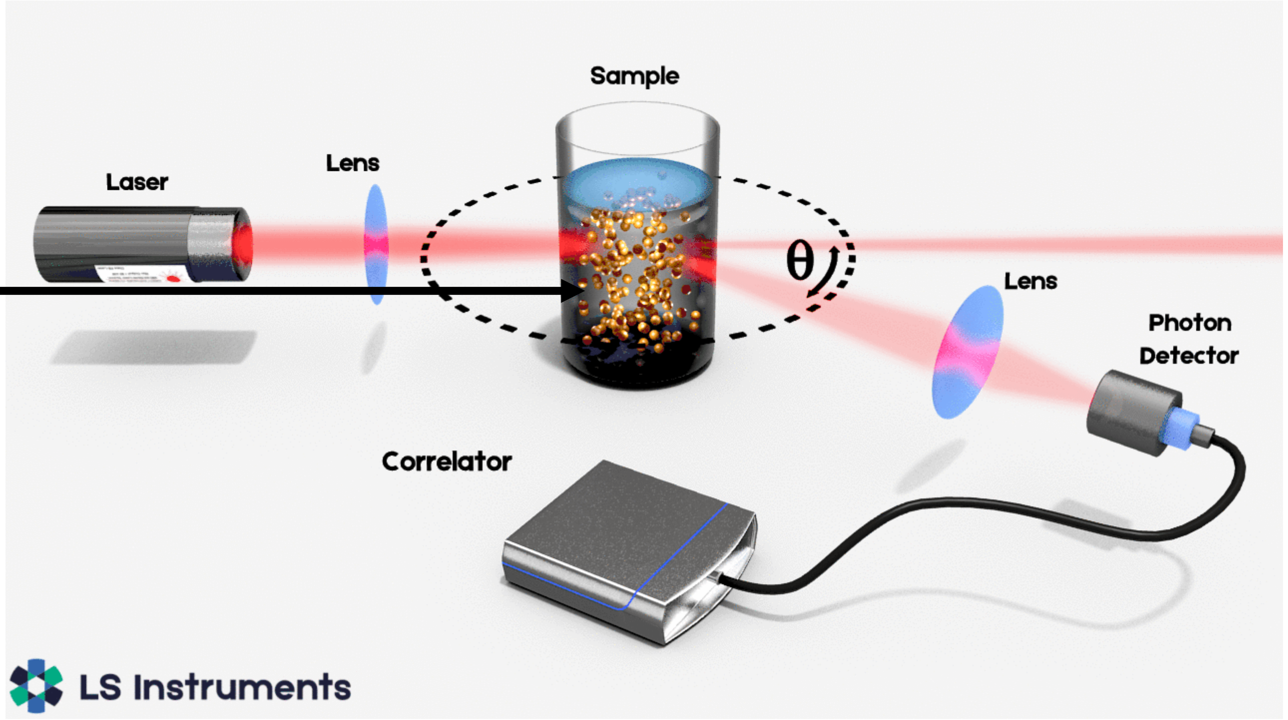
Pine et al., PRL 1988

Leheny et al., Curr Op Colloid Int Sci 2012



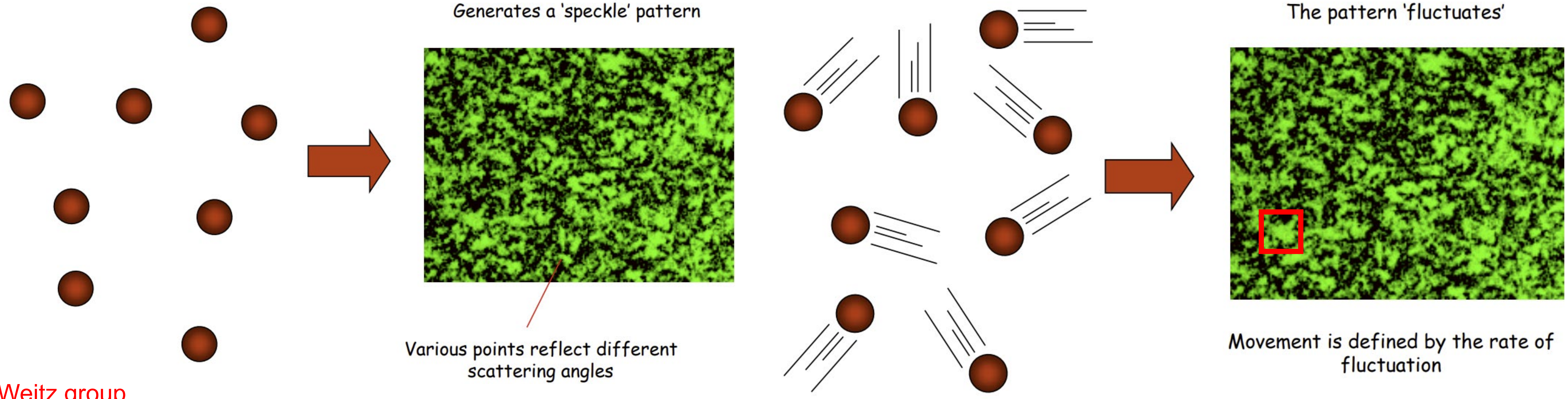
polystyrene microspheres





Pros:
Easier data analysis
Better ensemble averaging
Can even track system without tracers
Cons:
Need specialized instruments, and either large volumes (DWS) or radiation tolerance (XPCS)

Scattering microrheology



Generates a 'speckle' pattern

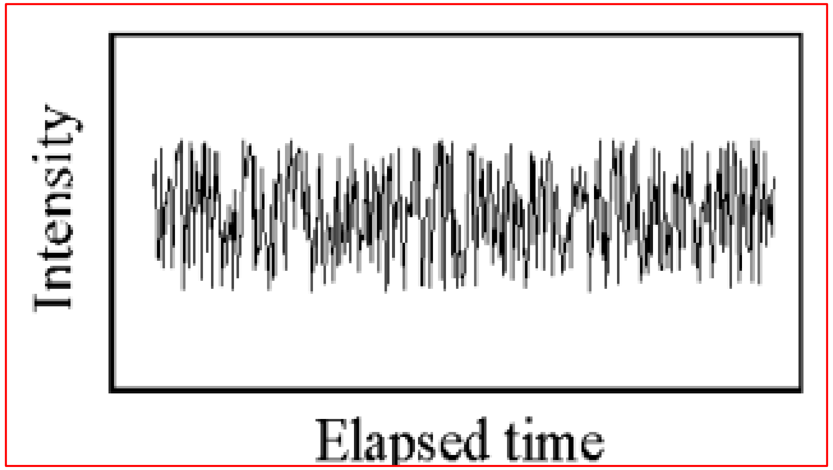
Various points reflect different scattering angles

The pattern 'fluctuates'

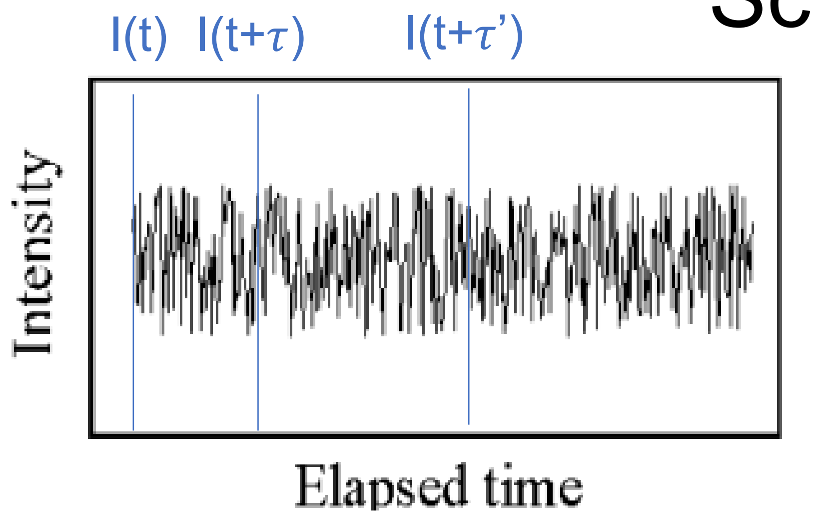
Movement is defined by the rate of fluctuation

Weitz group

Measure intensity fluctuation of speckles



Scattering microrheology

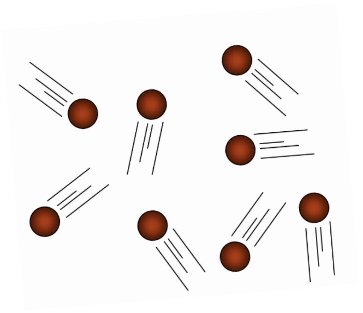


Intensity correlation function: integral of product of intensities at different delay times

$$g_2(t) = \frac{1}{t_{max}} \int_0^{t_{max}} I(t)I(t+\tau)d\tau$$

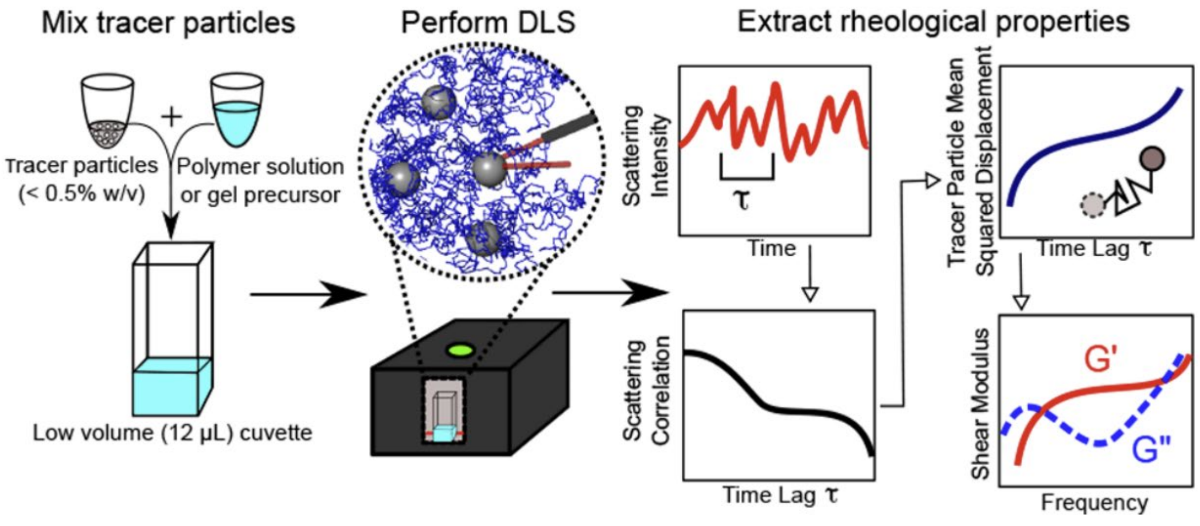
Now use Siegert relation: $g_2(\tau) = 1 + B|g_1(\tau)|^2$

Where g_1 is the field correlation (which describes correlated particle movement) and for Brownian motion follows:



$$g_1(\tau) = \exp\left(-\frac{q^2\Delta r^2(\tau)}{6}\right)$$

Once you have the MSD, use GSER to obtain $G^*(\omega)$



Krajina et al., ACS Cent Sci 2017